GYROSYNCHROTRON RADIATION CODE

DOCUMENTATION

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TABLE OF CONTENTS

Introduction & Summary

The core subroutine GETGSE and its subprograms compute the gyrosynchrotron emission and absorption coefficients. The main program GYROSPEC uses GETGSE to compute the emission and absorption coefficients in both the ordinary and extraordinary modes of polarization over a range of radio frequencies. Radio intensity, flux, brightness temperature, and fractional polarization are also computed for a homogeneous source region.

The gyrosynchrotron radiation formulae are from Ramaty, R., The Astrophysical Journal, Vol. 158, p. 753, 1969. The correction of Trulsen, J., and Fejer, J. A., Journal of Plasma Physics, 4, 825, 1970, is included. The program determines which cyclotron harmonics contribute to the gyrosynchrotron radiation for a given set of input parameters. Each harmonic is numerically integrated, starting with the lowest, and summed with the lower contributing harmonics (see "Gyrosynchrotron Radiation Formulae"). The inclusion of higher harmonics is halted (in subroutine GETJK) when their contribution falls below the relative error, RERR, specified by the user.

All input values are specified in GYROSPEC. The output is written to the file "output.txt". Another output file, "error.txt", provides information about errors detected by the code (see "Error Return Codes").

The subroutine DISTRN in the file "fdblpl.for" provides a normalized double-power-law electron distribution function and its derivative. The number density of suprathermal electrons is provided by the variable DENSTY.

To run the program, the following files must be compiled and linked:

gyrospectrum.for
getgse.for
dcadre.for
dmlin.for
mmbsjn.for
fdblpl.for
getgse.inc

Getgse.inc is an include file that specifies the values of some physical parameters and the variables in three common blocks (see "Labeled Common Blocks and Their Contents"). The include file allows these parameters and variables to be used throughout the code.

The IDL routine GYROPLOT.PRO reads and plots the output from GYROSPEC as a function of radio frequency. Plots of radio flux, brightness temperature, fractional polarization, and the emission and absorption coefficients as a function of radio frequency are displayed in four separate windows.

Contacts & Acknowledgements

This gyrosynchrotron code was originally developed in 1986 by Gordon D. Holman, NASA/Goddard Space Flight Center, and Thomas J. Kelly and Mark E. Pesses, then at Applied Research Corp. in Landover, Maryland. It has subsequently been revised by Gordon Holman.

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Gyrosynchrotron Radiation Formulae

The radiation intensity from a ***homogeneous*** source region is

$$I_{\pm}(\nu,\theta) = \frac{j_{\pm}}{K_{\pm}}[1-\exp(-K_{\pm}L)] \qquad \text{ergs (sec steradian Hz cm}^2)^{-1} \ .$$

Here j is the emissivity, K is the absorption coefficient, L is the depth of the source, ν is the observation frequency, θ is the angle between the wave normal and the magnetic field direction, and + (-) refers to the ordinary (extraordinary) mode of polarization. (Note that this is per unit *source* area.)

For an electron distribution function of the form Nf(γ,φ), where N is the number density of radiating electrons, γ is the relativistic factor $1/\sqrt{(1-\beta^2)}$, φ is the electron pitch angle and

$$2\pi \int_1^{\infty} d\gamma \int_{-1}^{1} d(\cos\phi)\, f(\gamma,\phi) = 1 \quad ,$$

the emissivity and absorption coefficient are given by

$$j_{\pm}(\nu,\theta) = 2\pi N \int_1^{\infty} d\gamma \int_{-1}^{1} d(cos\phi)\, f(\gamma,\phi)\, \eta_{\pm}(\nu,\theta,\gamma,\phi)$$

$$K_{\pm}(\nu,\theta) = \frac{2\pi}{m\nu^2 n_{\pm}} N \int_1^{\infty} d\gamma \int_{-1}^{1} d(\cos\phi)\eta_{\pm}(\nu,\theta,\gamma,\phi)\times$$

$$\left[-\beta\gamma^2 \frac{\partial}{\partial\gamma}\left(\frac{f(\gamma,\phi)}{\beta\gamma^2}\right) + \frac{n_{\pm}\beta\cos\theta - \cos\phi}{\beta^2\gamma\sin\phi} \frac{\partial}{\partial\phi} f(\gamma,\phi) \right] \quad ,$$

where $n_{\pm}$ is the index of refraction (given below) and $\eta_{\pm}$ is the gyrosynchrotron emissivity for a single electron:

$$\eta_{\pm}(\nu,\theta,\gamma,\phi) = \frac{2\pi e^2}{c}\nu^2 \sum_{s=-\infty}^{\infty} \frac{n_{\pm}}{1+a_{\theta_{\pm}}{}^2} \left[a_{\theta_{\pm}} \left(\frac{\cot\theta}{n_{\pm}} - \beta\frac{\cos\phi}{\sin\theta} \right) J_s(x_s) - \beta\sin\phi\, J_s'(x_s) \right]^2 \times$$

$$\times\delta\left(\nu - \frac{s\nu_B}{\gamma} - n_{\pm}\nu\beta\cos\phi\cos\theta \right) \quad .$$

Here,

$$x_s = \frac{s n_{\pm} \beta \sin\phi \sin\theta}{1 - n_{\pm}\beta\cos\phi\cos\theta} \quad ,$$

a_θ is the θ polarization coefficient (given below), J_s is a Bessel function of order s, and J_s' is the derivative of the Bessel function with respect to its argument. Note that $\nu_B = eB/(2\pi mc)$ and $\nu_p = e\sqrt{(n/\pi m)}$. For ν greater than the plasma cutoff frequencies ν_p and ν_x, s > 0.

$$a_{\theta_\pm}(\nu,\theta) = -\frac{2\nu\left(\nu_p{}^2 - \nu^2\right)\cos\theta}{-\nu^2\nu_B\sin^2\theta \pm \left[\nu^4\nu_B{}^2\sin^4\theta + 4\nu^2\left(\nu_p{}^2 - \nu^2\right)^2\cos^2\theta\right]^{1/2}}$$

$$n_\pm{}^2(\theta) = 1 + \frac{2\nu_p{}^2\left(\nu_p{}^2 - \nu^2\right)}{\pm\left[\nu^4\nu_B{}^4\sin^4\theta + 4\nu^2\nu_B{}^2\left(\nu_p{}^2 - \nu^2\right)^2\cos^2\theta\right]^{1/2} - 2\nu^2\left(\nu_p{}^2 - \nu^2\right) - \nu^2\nu_B{}^2\sin^2\theta}$$

Because of the presence of the Dirac delta function, one of the integrals can be done analytically. For θ ***not*** equal to π/2, the results are:

$$j_\pm(\nu,\theta) = \frac{4\pi^2 e^2 \nu N}{|\cos\theta|\, c}\frac{1}{\left(1 + a_{\theta_\pm}{}^2\right)}\int_1^\infty d\gamma \sum_{s_{\min}}^{s_{\max}} f(\gamma,\phi_s)\beta^{-1}$$

$$\times\left[a_{\theta_\pm}\left(\frac{\cot\theta}{n_\pm} - \beta\frac{\cos\phi_s}{\sin\theta}\right)J_s(x_s) - \beta\sin\phi_s\, J_s'(x_s)\right]^2$$

$$K_\pm(\nu,\theta) = \frac{4\pi^2 e^2 N}{mc\nu|\cos\theta|}\frac{1}{n_\pm\left(1 + a_{\theta_\pm}{}^2\right)}\int_1^\infty d\gamma \sum_{s_{\min}}^{s_{\max}} \beta^{-1} \times$$

$$\left[-\beta\gamma^2 \frac{\partial}{\partial\gamma}\left(\frac{f(\gamma,\phi)}{\beta\gamma^2}\right)+\frac{n_{\pm}\beta\cos\theta-\cos\phi}{\beta^2\gamma\sin\phi}\frac{\partial}{\partial\phi}f(\gamma,\phi)\right]_{\phi=\phi_s} \times$$

$$\left[a_{\theta_{\pm}}\left(\frac{\cot\theta}{n_{\pm}}-\beta\frac{\cos\phi_s}{\sin\theta}\right)J_s(x_s)-\beta\sin\phi_s\,J_s'(x_s)\right]^2 ,$$

where

$$\cos\phi_s=\frac{1-\frac{s\nu_B}{\gamma\nu}}{n_{\pm}\beta\cos\theta} \quad , \qquad x_s=\frac{sn_{\pm}\beta\sin\theta\sin\phi_s}{1-n_{\pm}\beta\cos\theta\cos\phi_s} \quad ,$$

$$s_{\min}=Integer\left[\frac{\gamma\nu}{\nu_B}\left(1-n_{\pm}\beta\cos\theta\right)\right]+1$$

$$s_{\max}=Integer\left[\frac{\gamma\nu}{\nu_B}\left(1+n_{\pm}\beta\cos\theta\right)\right] \quad .$$

There is an alternative approach to computing j and K that allows the summation over s to be taken outside of the integral. This approach is used in GETGSE. The expressions for j and K are the same as above, but the integration over γ is from γ_1 to γ_2 and the summation is from a minimum value of s to s = ∞:

$$s>\left(\frac{\nu}{\nu_B}\right)\left(1-n_{\pm}^{\,2}\cos^2\theta\right)^{1/2}$$

$$\gamma_{1,2}=\frac{\left(\frac{s\nu_B}{\nu}\right)^2+n_{\pm}^{\,2}\cos^2\theta}{\left(\frac{s\nu_B}{\nu}\right)\pm n_{\pm}\left|\cos\theta\right|\left[\left(\frac{s\nu_B}{\nu}\right)^2+n_{\pm}^{\,2}\cos^2\theta-1\right]^{1/2}}$$

When θ = π/2, the following expressions are obtained for j and K:

$$j_{\pm}\left(\nu,\frac{\pi}{2}\right)=\frac{4\pi^2e^2N}{c}\nu_B\,n_{\pm}\sum_{s=1}^{\infty}\beta_s^2\int_{-1}^{1}d(\cos\phi)\,f(\gamma_s,\phi)Y_{\pm}(\phi)\quad ,$$

$$K_{\pm}\left(\nu,\frac{\pi}{2}\right)=\frac{4\pi^2e^2N}{mc\nu^2}\nu_B\sum_{s=1}^{\infty}\beta_s^2\int_{-1}^{1}d(\cos\phi)\,Y_{\pm}(\phi)$$

$$\times\left[-\beta\gamma 2\,\frac{\partial}{\partial\gamma}\left(\frac{f(\gamma,\phi)}{\beta\gamma 2}\right)+\frac{n\beta\cos\theta-\cos\phi}{\beta 2\gamma\sin\phi}\,\frac{\partial}{\partial\phi}\,f(\gamma,\phi)\right]_{\gamma=\gamma_s,\,\beta=\beta_s}$$

where

$$Y_{+}(\phi)=\cos^2\phi[J_s(x_s)]^2\ ,\qquad Y_{-}(\phi)=\sin^2\phi[J_s'(x_s)]^2\ ,$$

$$\gamma_s=\frac{s\nu_B}{\nu}\quad ,\qquad \beta_s=\left(1-\gamma_s^{-2}\right)^{1/2}\quad ,\qquad x_s=s\beta_s n_{\pm}\sin\phi\quad .$$

The frequency ν should always be greater than ν_p for the ordinary (+) mode, and greater than

$$\nu_x=\left(\nu_p{}^2+\left(\frac{1}{4}\right)\nu_B{}^2\right)^{1/2}+\left(\frac{1}{2}\right)\nu_B$$

for the extraordinary (-) mode. The extraordinary mode emission will usually dominate.

A typical electron distribution function is the isotropic power law:

$$f(\gamma,\phi)=\frac{(p-1)}{4\pi}(\gamma_{\min}-1)^{p-1}\,(\gamma-1)^{-p}\qquad \gamma\geq\gamma_{\min}$$

$$=0\qquad \gamma<\gamma_{\min}$$

Good test values for the parameters are $\gamma_{min} = 1.2$, $p = 3$, $\theta = 45^{\circ}$, and $\nu_p/\nu_B = 0.5$ and 3.

General Code and Programming Practices

1) All program units have IMPLICIT NONE statements. Therefore, all variables and parameters are type declared. This ensures also against typing errors in the code.

2) All real variables and parameters are REAL*8 (double precision).

3) All integer variables and parameters are INTEGER*4.

4) Six-character variable and subprogram names are frequently used throughout the program.

5) CGS units are used throughout.

6) All mathematical and physical constants are declared as parameters, and are available in every program unit that uses the GETGSE.INC include file.

7) All input parameters are specified in the main program, GYROSPEC. A unit number for reading is specified by IUNITA. Here, IUNITA = 5. This means that the data is to be read from the user's terminal keyboard.

8) All output statements use the WRITE statement. A unit number for output is specified by IUNITB. Here, IUNITB = 6, and that is the number of the user's terminal screen.

9) This code uses explicitly three subprograms based on subprograms found in Numerical Recipes in FORTRAN 77, Second Edition, (Cambridge University Press). The routines are DMLIN, and DCADRE, which evaluate the integrals numerically, and MMBSJN, which evaluates the Bessel functions and their derivatives.

10) DMLIN solves the integrals via the method of Gaussian Quadrature. DMLIN will use up to 256 points to estimate the value of the integral. If DMLIN cannot get a result with the absolute or relative error requirements of the user, then it returns with an error code equal to 120.

11) DCADRE solves the integrations via the adaptive Romberg extrapolation method. This routine subdivides the interval between limits of integration, until the change in error estimate is within the error requirements.

12) Input and calculated values needed by the routines called by DMLIN and DCADRE are passed through the labeled common block INPUT. These variables are listed in "Labeled Common Blocks and their Contents".

13) MMBSJN evaluates Bessel functions and their derivatives of the first kind of non-negative integer order for real arguments. All the calls to MMBSJN are made in the routine GETBES. A call to MMBSJN returns the values of evaluated Bessel functions, BESSL, and their derivatives, DBESSL.

14) An error return code of zero means NO error has occurred. A non-zero value means an error has occurred. The program aborts after all error codes, except for some from the calculations of the Bessel functions by MMBSJN. These particular error return codes show that the value of some Bessel functions are smaller than the smallest, non-zero, positive number on the machine (here, approximately 1.D-38), and therefore some Bessel functions have been set to zero.

General Execution of Program and Annotated Notes of Gyrosynchrotron Radiation Formulas

ROUTINES (and Subprograms called)	Description of routine
GYROSPEC (calls GETGSE)	All input values to the program are entered in GYROSPEC. Computed results are written to the file output.txt.
GETGSE (calls INITLZ & GETJK)	This is the routine called to solve the summing and integration. In other words, this is the front-end routine that the user communicates with. The routine INITLZ is called to get values of quantities that are not affected by the integration and summing, and GETJK is in charge of the actual summing and integration.
INITLZ (calls GETIND, GETATH, SLIM90, & SLIMIT)	The routine INITLZ initializes all unchanging values needed throughout the program. Through a call to GETATH, it gets A-THETA. Through a call to GETIND, it gets the index of refraction. The limits of summing over the harmonics, NSMIN & NSMAX, i.e., the minimum and maximum limits, are obtained from a call to SLIM90 ($\theta = 90$), or SLIMIT ($\theta \neq 90$). INITLZ also determines the cosine and sine of THETA.
GETJK (calls GET90 & GETEMI)	GETJK governs the summing and evaluation of the integrals. GETJK sums the value of the integrals for the emissivity, j, and the absorption coefficient, K. If THETA = 90 degrees, then GETJK calls GET90, otherwise, GETEMI is called.
GET90 & GETEMI (calls DMLIN or DCADRE)	GET90 and GETEMI call the routines DMLIN or DCADRE, depending upon the method of integration chosen. DMLIN is Gaussian Quadrature, DCADRE is adaptive Romberg.

DMLIN (calls E90FCN & A90FCN, for $\theta = 90$ degrees)	DMLIN uses the Gaussian Quadrature method to numerically estimate integrals. For $\theta = 90$ degrees, DMLIN calls E90FCN for the value of the emissivity (j) integrand, and A90FCN for the absorption coefficient (K) integrand. These integrands are dependent on NS, the index of the harmonic.
DMLIN (calls EMIFCN & ABSFCN, for $\theta \neq 90$ degrees)	Same as for DMLIN above, except $\theta \neq 90$ degrees. The function EMIFCN gets the j integrand, and ABSFCN gets the value of the K integrand, and both integrands depend on NS.
DCADRE (calls E90ROM & A90ROM, for $\theta = 90$ degrees)	DCADRE uses the Romberg method of numerical integration. For $\theta = 90$ degrees, DCADRE calls E90ROM for the value of the emissivity (j) integrand, and A90ROM for the absorption coefficient (K) integrand. These integrands are dependent on NS, the index of the harmonic.
DCADRE (calls EMIROM & ABSROM, for $\theta \neq 90$ degrees)	Same as for DCADRE above, except $\theta \neq 90$ degrees. The function EMIROM gets the j integrand, and ABSROM gets the value of the K integrand, and both integrands depend on NS.
E90FCN & A90FCN (calls GETBES & DISTRN)	For THETA = 90 degrees, these two functions evaluate the integrands, as a function of the cosine of PHI, COSPHI. The Bessel function and its first derivative are obtained from GETBES, and the distribution function and its derivatives are obtained from DISTRN.
EMIFCN & ABSFCN (calls GETPHI, GETXS, INTGND, & DISTRN)	For THETA not equal to 90 degrees, these two functions evaluate the integrands as a function of GAMMA. Both EMIFCN & ABSFCN call 1) GETPHI, to get value of the cosine of PHI-S, CPHIS, and the sine, SPHIS, 2) GETXS, to get the value of XS, 3) INTGND, to evaluate the squared term, common to the integrands of both j & K, and 4) DISTRN, to get the value and first derivatives in GAMMA and PHI of the distribution function.

E90ROM & A90ROM (calls E90FCN & A90FCN, resp.)	For THETA = 90 degrees, these two functions get the value of the integrand for DCADRE. E90ROM & A90ROM just call E90FCN & A90FCN, resp., and transfer the values from E90FCN & A90FCN back to DCADRE.
EMIROM & ABSROM (calls EMIFCN & ABSFCN, resp.)	For THETA not equal to 90 degrees, these two functions get the value of the integrand for DCADRE. EMIROM & ABSROM just call EMIFCN & ABSFCN, resp., and transfer the values from EMIFCN & ABSFCN back to DCADRE.
GETBES (calls MMBSJN)	GETBES calls MMBSJN to evaluate the Bessel function and its derivative to order NS.
DISTRN (no calls)	User supplied subroutine to calculate the distribution function and its first derivatives in GAMMA and PHI. Note that the distribution function is normalized so that the integral over GAMMA and PHI is one. The total density of radiating electrons is given by the variable DENSTY.
SLIMIT (calls GAMLIM)	Function that determines the range of harmonics to include in the summation of integrals for $\theta \neq 90^{\circ}$.
SLIM90 (no calls)	Function that determines the range of harmonics to include in the summation of integrals for $\theta = 90^{\circ}$.

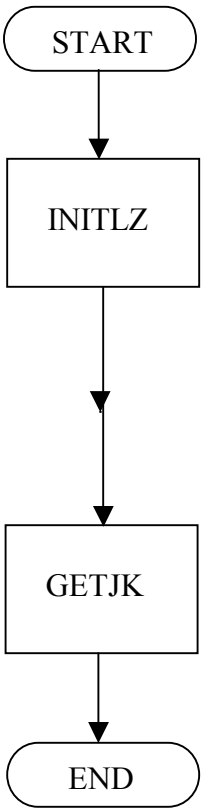
Integrands for Gaussian Integration determined by routines

	$\theta \neq 90^\circ$	$\theta = 90^\circ$
j	EMIFCN	E90FCN
K	ABSFCN	A90FCN

Integrands for Romberg Integration determined by routines

	$\theta \neq 90^\circ$	$\theta = 90^\circ$
j	EMIROM	E90ROM
K	ABSROM	A90ROM

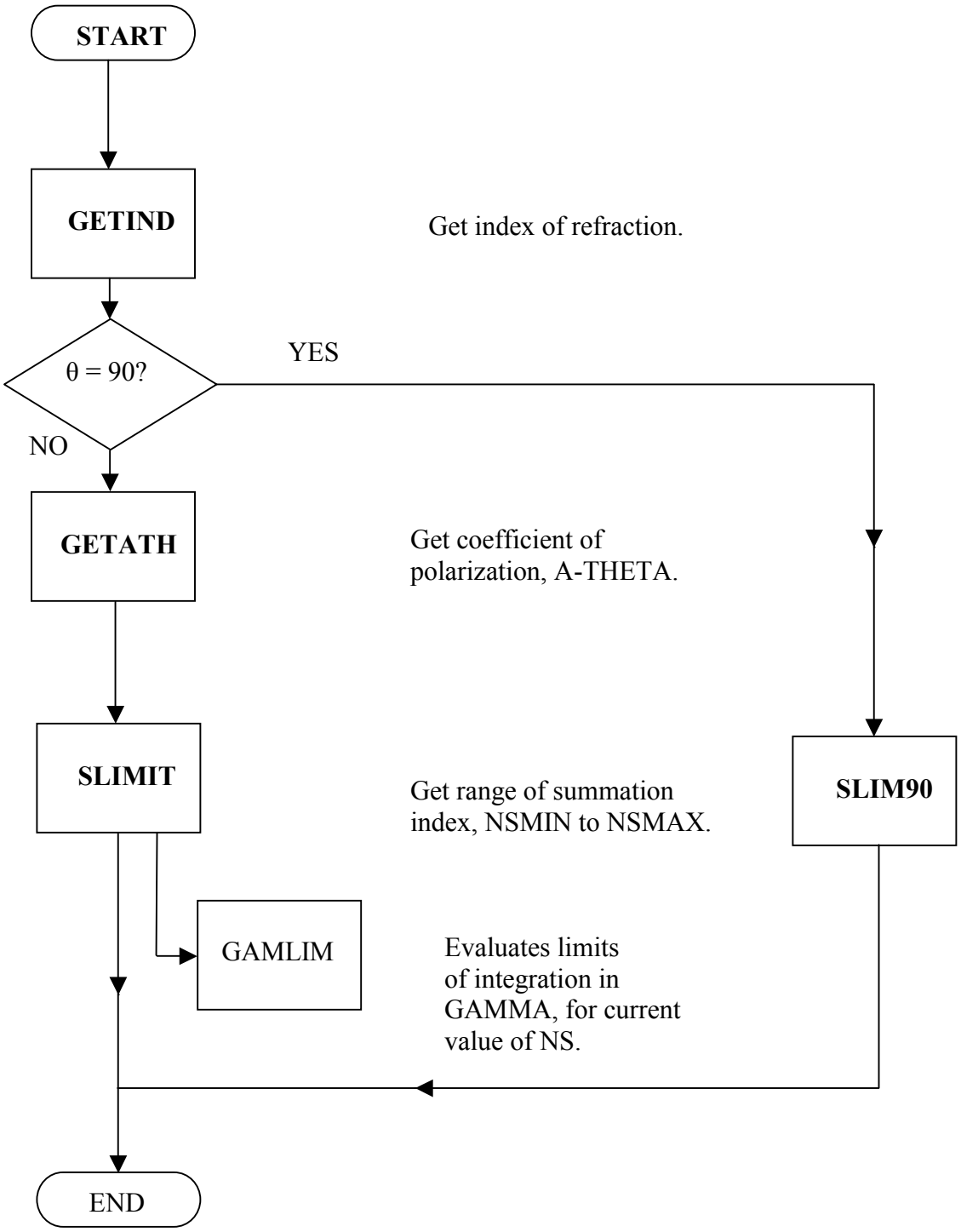
Flowchart of Program Unit GETGSE



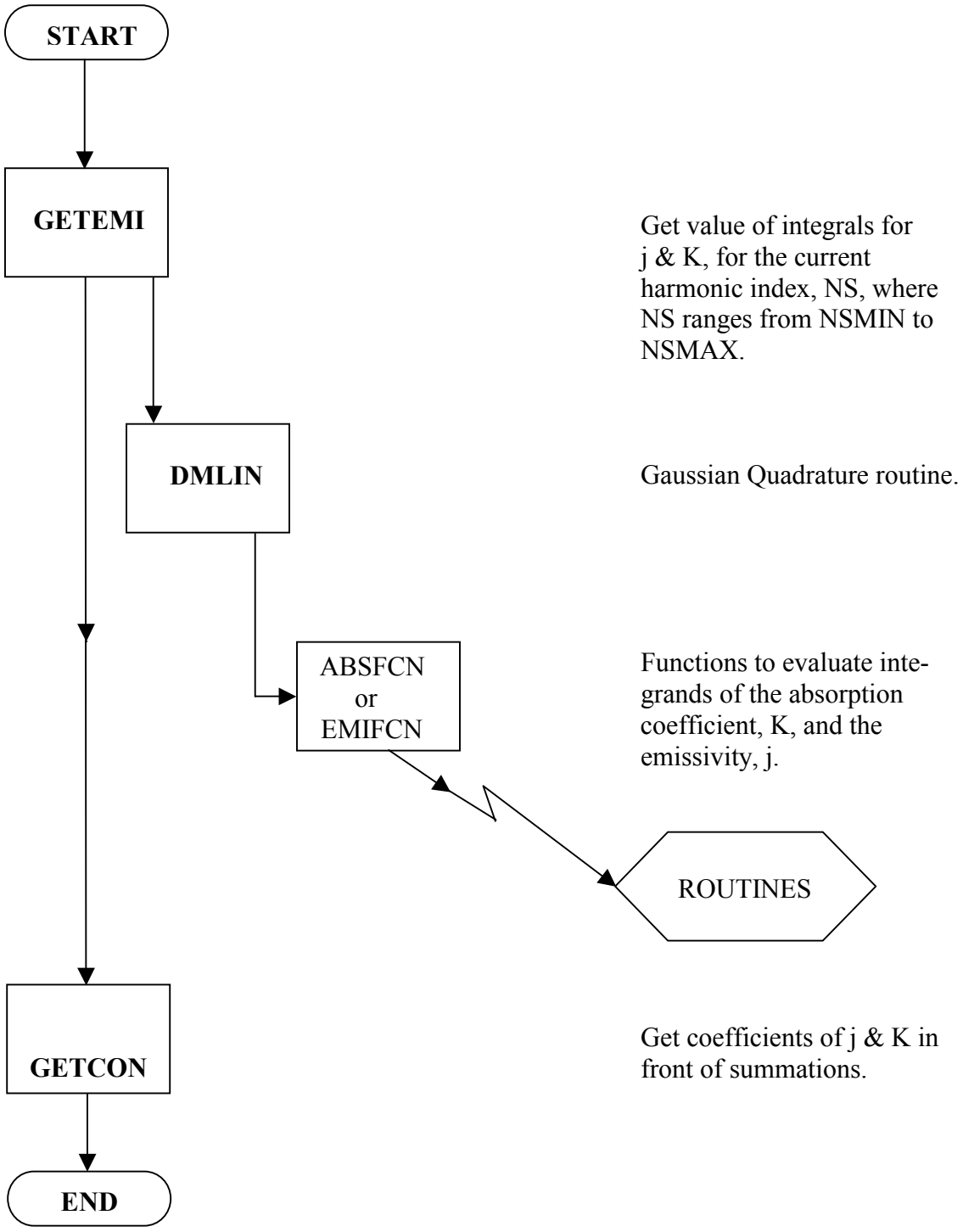
Evaluates
1) index of refraction,
2) polarization coefficient,
3) cosine and sine of THETA, and
4) NSMIN & NSMAX, limits of summation.
Calls several subprograms.

Solves integrals for j & K, with calls to DMLIN or DCADRE , then sums the harmonics.

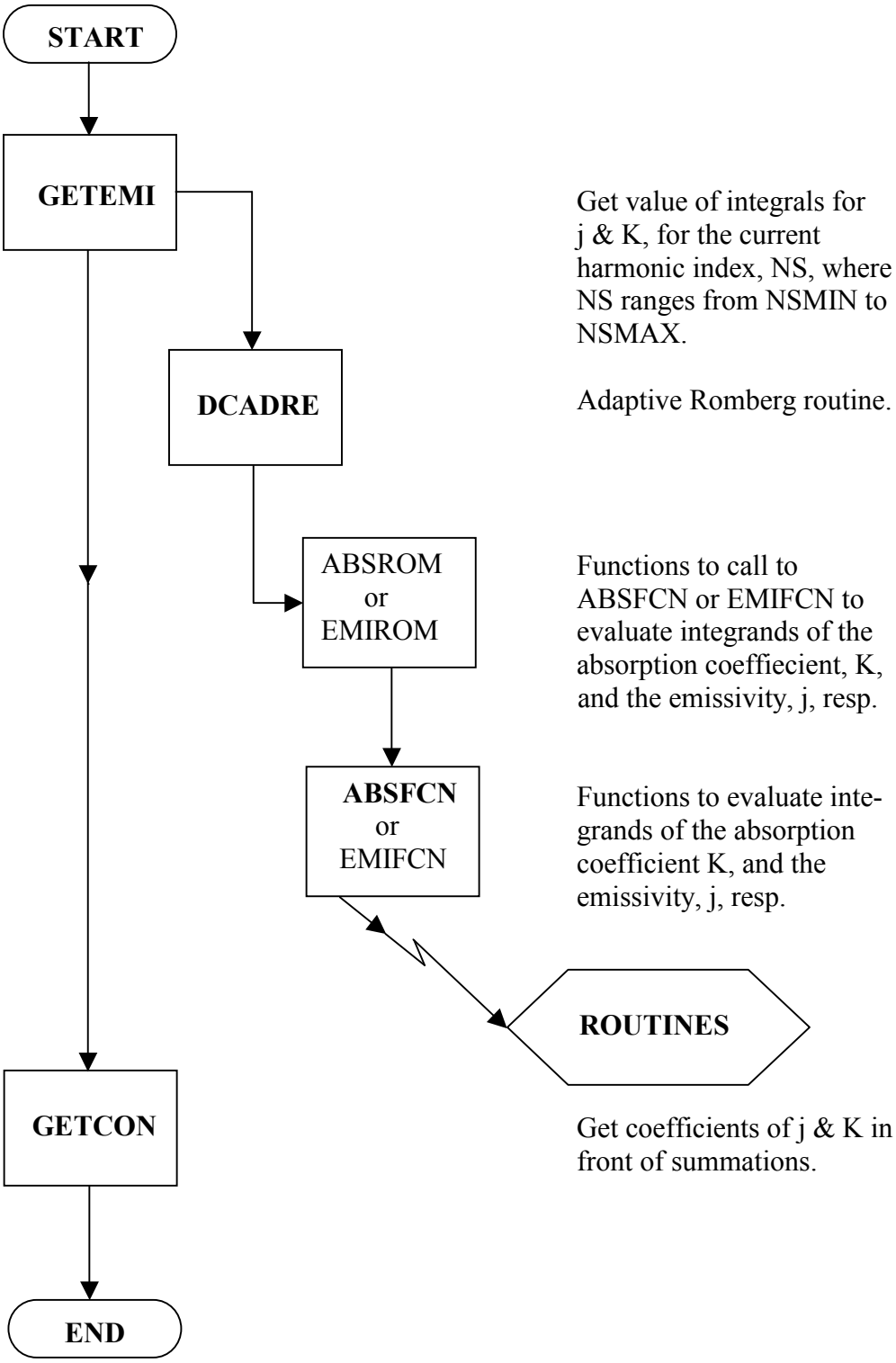
Flowchart of subroutine INITLZ



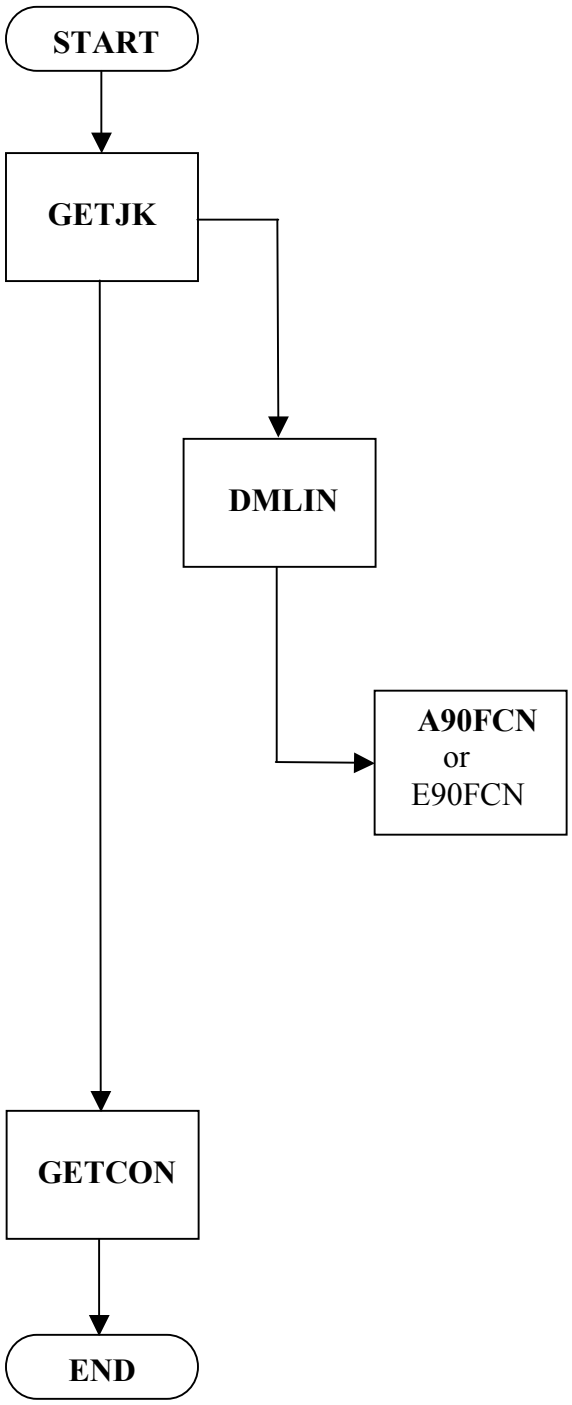
Flowchart of GETJK for θ ≠ 90 Degrees: Gaussian Quadrature Method



Flowchart of GETJK for θ ≠ 90 Degrees: Cautious, Adaptive Romberg Method



Flowchart of GETJK for θ = 90 Degrees: Gaussian Quadrature Method



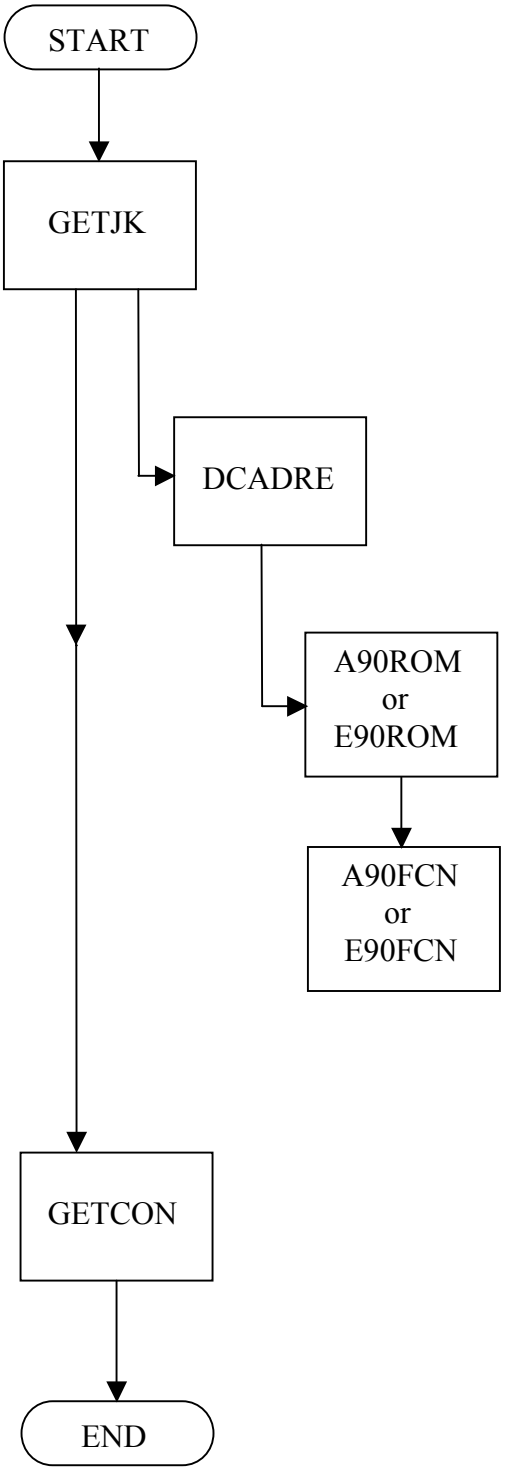
Get value of integral of order NS, for summation of J & K, θ = 90 degrees.

Gaussian Quadrature routine.

Functions to evaluate integrands of the absorption coefficient, K, and the emissivity, j, at 90 deg. Both routines call GETBES for values of Bessel function and its first derivative, of order NS.

Get coefficients of j & K in front of summations.

Flowchart of GETJK for θ = 90 Degrees: Cautious, Adaptive Romberg Method



Get value of integral of order NS, for summation of j & K, θ = 90 degrees.

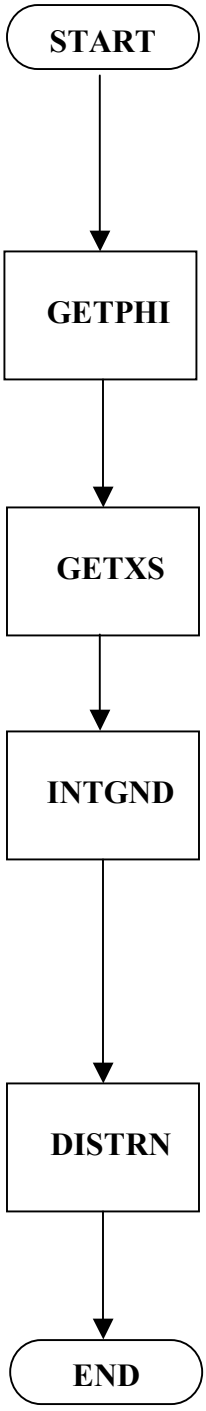
Adaptive Romberg routine.

Functions to call routines to evaluate integrands for j & K.

Functions to evaluate integrands of the absorption coefficient, K, and the emissivity, j, at 90 deg. Both routines call GETBES for values of Bessel function and its first derivative, of order NS.

Get coefficients of j & K in front of summations.

Flowchart of Routines Called by ABSFCN & EMIFCN



Get values of PHI-S, and the cosine and sine of PHI-S, CPHIS, & SPHIS, resp.

Get value of XS.

Get value of the term that is squared in the integrands for both j & K. INTGND calls GETBES, for values of the Bessel function and its first derivative, of order NS.

Get values of the distribution function and the first derivative in GAMMA and PHI. This routine is user supplied.

Input and Output Variables

INPUT

Variable names	Variable descriptions
GAMLOW, GAMHIGH	Range of GAMMA over which to integrate.
GAMBRK	Break energy in double-power-law electron distribution
MODE	Mode of wave polarization. 1 for Ordinary and -1 for Extraordinary.
BFIELD	Magnetic field strength in gauss.
NTH	Thermal plasma number density.
RNU, RNUB, RNUP	Frequencies: Wave, cyclotron, plasma.
THETAD	Angle between the wave normal and the magnetic field direction.
DENSTY	Number density of radiating electrons.
P	Lower power-law index of electron distribution function.
Q	Upper power-law index of electron distribution function.
D	Thickness of source region in cm.
METHOD	Method of integration, either Gaussian Quadrature (1) or Adaptive Romberg (2) or both (3).
RERR	Desired relative error for routines to evaluate the integrals. The relative error parameter RERR must be in the interval (0.0, 0.1) inclusively. For example, RERR = 0.01 indicates that the estimate of the integral is to be correct to one digit, whereas RERR = .0001 calls for fours digits of accuracy. If DCADRE determines that the relative accuracy requirements cannot be satisfied, IERROM is set to 100 (RERR should be large enough that, when added to 100.0, the result is a number greater than 100.0, i.e., beware of round off error with very small error requirements).

OUTPUT

Variable names	Variable descriptions
VALUEJ	Solution to emissivity calculation, J.
VALUEK	Solution to absorption coefficient calculation, K.
IERGQ	Evaluation of integral via Gaussian Quadrature method by DMLIN. 120 (fatal error) more than 256 Gauss points would be needed to obtain the desired accuracy. May help to attempt a solution by partitioning between limits of GAMMIN and GAMMAX.
IERROM	Evaluation of integral via cautious adaptive Romberg method by DCADRE. 100 (fatal error) indicates that RERR is greater than 0.1, or RERR is less than 0.0. 101 indicates that that the convergence criteria has not been met after the maximum number of iterations were performed in the series. 102 means that there was a failure in the subroutine polint.

Error Return Codes

Error return code variable	Routines	Description/Meaning
IERNU	INITLZ	Test that wave frequency is above the wave mode cutoff frequency. l means the frequency of the ordinary mode wave is below cutoff -1 means the frequency of the extraordinary mode wave is below cutoff
IERIND	GETIND	Calculation of index of refraction -1 means denominator = 0 -2 square root of negative number
IERATH	GETATH	Calculation of coefficient of polarization -l means denominator = 0
IERNS	SLIMIT	Determination of NSMIN & NSMAX. θ not equal to 90 degrees: -l square root of # < 0 -2 NSMIN > NDIMEN-2 -3 NSMAX > NDIMEN-2
	SLIM90	θ = 90 degrees: 1 NSMIN > NDIMEN-2 2 NSMAX > NDIMEN-2
IERGAM	GAMLIM	Calculation of limits of integration in GAMMA, for some value of NS, the index of summation, when THETA not equal to 90 degrees. -1 square root of number < 0 -2 denominator = 0 in getting GAMMAI -3 denominator = 0 in getting GAMMAF
IEREMI	GETJK	If IERGQ reports an error, IEREMI records which term in the summation caused the error. If IEREMI < 0, then the error was due to evaluation of a j term, and IEREMI > 0 indicates the error was due to evaluation of a K term

Error return code variable	Routines	Description/Meaning
IERPHI	GETPHI	Calculation of cosine and sine of PHI-S. For THETA ≠ 90 degrees only. -1 means denominator = 0 -2 means magnitude of cos(PHI-S) > 1
IERBES	GETBES	Calculation of Bessel function. 140 means argument, XS, or order, NS, became out of range. 141 indicates that the convergence criteria has not been met after maximum number of iterations have been performed in the series. 142 indicates that complex continued fraction cf2 failed to converge after the maximum number of iterations have been performed.
IERGQ	DMLIN	Evaluation of integral via Gaussian Quadrature method . 120 (fatal error) more than 256 Gauss points would be needed to obtain the desired accuracy. May help to attempt a solution by partitioning between limits of GAMMIN and GAMMAX.
IERROM	DCADRE	Evaluation of integral via cautious adaptive Romberg method. 100 (fatal error) indicates that RERR is greater than 0.1, or RERR is less than 0.0, or RERR is too small for the precision of the machine. 101 indicates that the convergence criteria has not been met after the maximum number of iterations have been performed in the series. 102 indicates that there is a failure in the subroutine polint.
IERXS	GETXS	Calculation of XS. For THETA ≠ 90 degrees. -1 means denominator = 0

Error return code variable	Routines	Description/Meaning
IERCAT	INTGND	Calculation of squared term, common to both j & K integrands. -1 means index of refraction = 0 -2 sin(THETA) = 0, but uncertainty in value of INTGND term. Expected to be rare.

Labeled Common Blocks and Their Contents

All variables except four are passed to GETGSE through labeled common blocks. Those 4 are the output values for (1) the emissivity, j, (2) the absorption coefficient, K, (3) an error return code to report on how the Gaussian Quadrature integration went, and (4) an error return code to report on how the Romberg integration went, whichever method of integration applies.

There are three labeled common blocks used by GETGSE. These common blocks are INPUT, INTRNL, and ERBLCK. Each common block has its own function. INPUT passes all input information to GETGSE. INTRNL passes internal working variables throughout GETGSE and the routines it will call, and ERBLCK reports the error condition codes from the calculations of the index of refraction, the polarization coefficient, etc.

INPUT Common block to pass input values to routines called DMLIN and DCADRE. The common block is

```
 COMMON   /INPUT/ MODE, METHOD,
1         RNU, RNUB, RNUP, P, Q, GAMMIN, GAMMAX, THETAD,
2         DENSTY, RERR
```

The variables and a description follow.

	Variable name	Variable description
1)	MODE	Mode of wave polarization, l for ordinary and -1 for extraordinary mode
2)	METHOD	Method of integration, Gaussian or Romberg, l for Gaussian, 2 for Romberg, 3 for both.
3)	RNU	Wave frequency, in GHz
4)	RNUB	Cyclotron frequency, in GHz
5)	RNUP	Plasma frequency, in GHz
6)	P, Q	Powers of the double power-law distribution function
7)	GAMMIN	Lower integration limit in GAMMA
8)	GAMMAX	Upper integration limit in GAMMA
9)	THETAD	Angle theta, in degrees, between the wave normal and the magnetic field direction.

10)	DENSTY	The density of radiating electrons, in number per cubic cm
11)	RERR	The relative error requirement for numerical integration

INTRNL Common block to pass internal working variables used throughout by GETGSE and all the routines it needs. The common block is

```
COMMON   /INTRNL/
1        NS, NSMIN, NSMAX, NSCON,
2        RINDX, ATHET, STHETA, CTHETA, ERROM, THETAR
```

and the variables and a description follows.

	Variable name	Variable description
1)	NS	Value of the harmonic summation index
2)	NSMIN	Minimum value of harmonics summed over
3)	NSMAX	Maximum value of harmonics summed over
4)	NSCON	Maximum harmonic summed for convergence within RERR (NSCON ≤ NSMAX)
5)	RINDX	Index of refraction
6)	ATHET	Value of coefficient of polarization
7)	CTHETA	Cosine of THETA
8)	STHETA	Sine of THETA
9)	ERROM	Estimate of error from DCADRE
10)	THETAR	Value of THETA in radians.

ERBLCK Common block to pass error return codes from routines called by GETGSE. ALL error returns, EXCEPT IERGQ (for Guassian Quadrature integration) AND IERROM (for Romberg integration) are passed through ERBLCK. The common block is

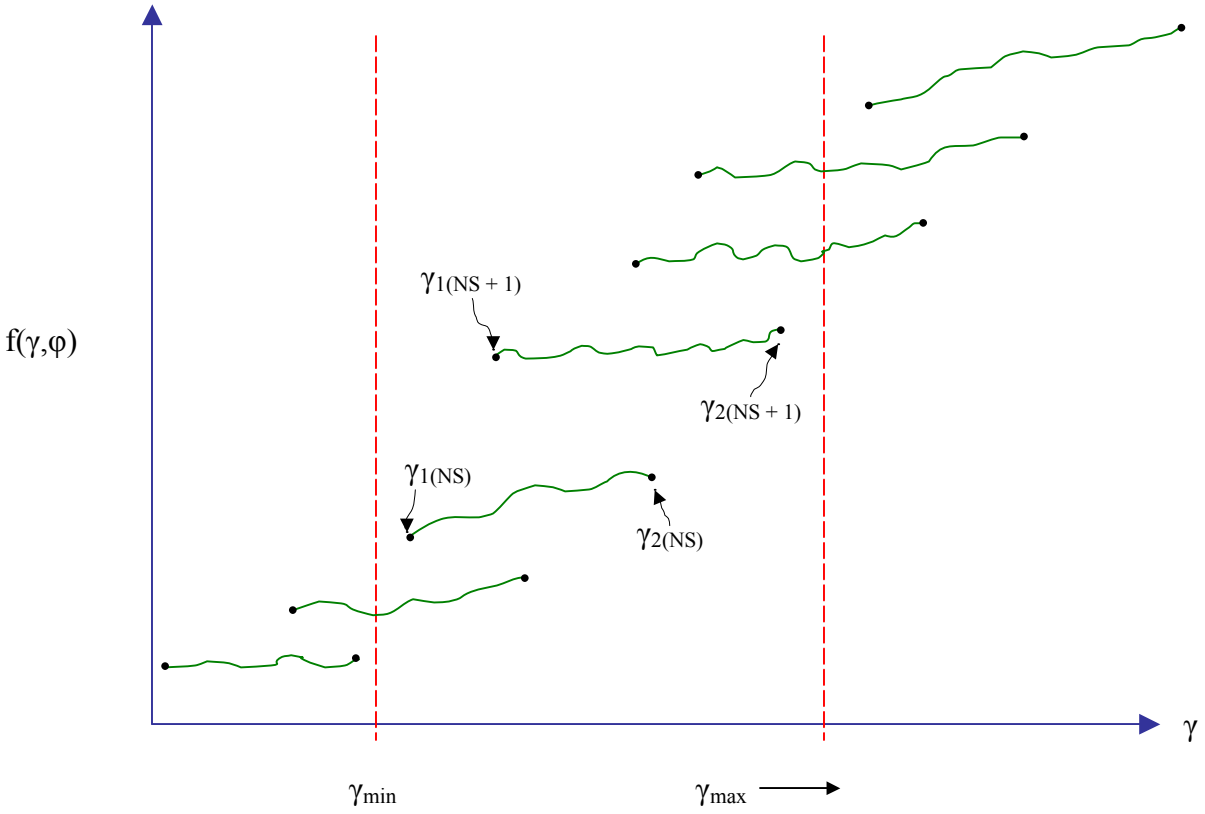
```
COMMON
1          /ERBLCK/
2          IERATH, IERBES, IERIND, IERCAT, IERXS,
           IERGAM, IEREMI, IERNS, IERNU, IERPHI
```

and the variables and a description follows. A more extensive description is included in section 4 on error return codes.

	Variable name	Variable description
1)	IERATH	Error return code from GETATH
2)	IERBES	Error return code from GETBES
3)	IERIND	Error return code from GETIND
4)	IERCAT	Error return code from INTGND
5)	IERXS	Error return code from GETXS
6)	IERGAM	Error return code from GAMLIM
7)	IEREMI	Error return code from GETEMI
8)	IERNS	Error return code from SLIM90 & SLIMIT
9)	IERNU	Error return code from INITLZ
10)	IERPHI	Error return code from GETPHI

How the Limits of the Summing Index, NS, are Determined, where NS is Summed from NSMIN to NSMAX

$$\sum_{NS = NSMIN}^{NS = NSMAX} \int_{Z1}^{Z2} \ldots dZ$$



Compare γ_1 and γ_2 for each NS, with γ_{min} and γ_{max}.

1) Find which value of NS is the lowest, such that $\gamma_2(NS) \geq \gamma_{min}$; this value for NS = NSMIN.
2) Find which value of NS is the largest, such that $\gamma_1(NS) \geq \gamma_{max}$; this value for NS = NSMAX + 1, or NSMAX = NS – 1.

Then the limits of NS are

$$\sum_{NS = NSMIN}^{NS = NSMAX} \int_{Max(\gamma_{min}, \gamma_1(NS))}^{Min(\gamma_{max}, \gamma_2(NS))} \ldots d\gamma \quad ,$$

and the summation includes all appropriate harmonics of NS.

Evaluation of $[J_s(x)/x]$ as $x \rightarrow 0$

From "Handbook of Mathematical Functions", by Abramowitz and Stegun, Dover Press, Ch. 9, pg. 361, 1964, we have the recurrence relation, from 9.1.27,

(1) $$\left(\frac{J_s(x)}{x}\right) = \frac{1}{2s}\left(J_{s-1}(x) + J_{s+1}(x)\right)$$

If we evaluate the limit, for s = 0, we obtain

(2) $$\underset{x\to 0^{\pm}}{Lim}\left[J_o(x)\Big/x\right] = J_o(0)\cdot Lim\left[1\Big/x\right] = \pm\infty \quad .$$

If we evaluate the limit, for s = 1, we obtain from (1)

(3) $$\underset{x\to 0^{\pm}}{Lim}\left[J_1(x)\Big/x\right] = \frac{1}{2}\left(J_o(0) + J_2(0)\right) = \frac{1}{2}J_o(0) \quad .$$

If we evaluate the limit, for s > 1, we obtain from (1)

(4) $$\underset{x\to 0^{\pm}}{Lim}\left[J_s(x)\Big/x\right] = \frac{1}{2s}\left(J_{s-1}(0) + J_{s+1}(0)\right) = 0\,,$$ since $J_s = 0$, for all values of s, $s \geq 1$.