EUV IMAGING SPECTROMETER
Hinode

EIS SOFTWARE NOTE No. 15

Version 2.3 8 July 2011

Deriving densities, column depths and filling factors from Hinode/EIS data

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1 Overview

The EIS instrument has access to some of the best coronal density diagnostics that allow good density estimates to be made from individual spatial pixels, thus enabling the creation of density maps. From a density map, a map of plasma column depths can then be produced which can then be used to estimate the plasma filling factor. This document describes methods and software for performing this work with EIS data.

The software described requires that the CHIANTI database (Dere et al., 1997, 2009) is part of your Solarsoft distribution.

2 Basics of density diagnostics

A *density diagnostic* is pair of emission lines emitted from the same ion for which the ratio of emissivities is sensitive to the electron density. Fig. 1 shows the variation of the emissivity ratio for a pair of Mg VII lines commonly used in EIS data analysis. The emissivity is a theoretical quantity that represents the energy released through radiation from a particular emission line from a unit volume of plasma. For this work the emissivities are computed using the CHIANTI atomic database and, in particular, the EMISS_CALC procedure in CHIANTI.

The detailed reasons for why particular line ratios are sensitive to density are beyond the scope of this document, and so interested readers are referred to Dere & Mason (1981) and Mariska (1992).

Most coronal lines are optically thin and so the measured line intensity is directly proportional to the emissivity, which means that the measured intensity ratio can be converted to a density using the emissivity ratio plot (such as that shown in Fig. 1). The software described in the present work assumes that the observed lines are optically thin.

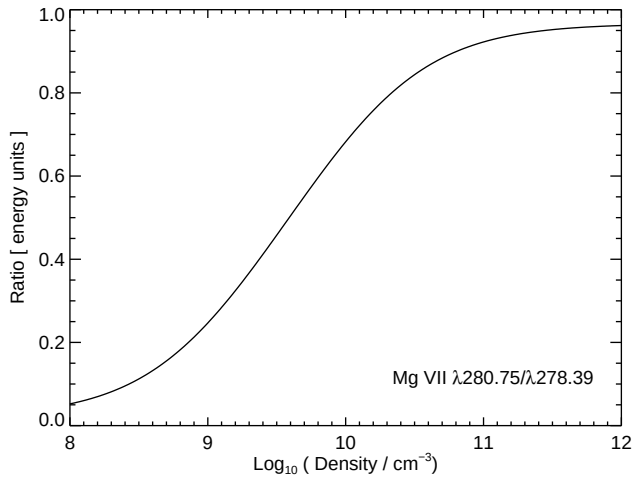

Figure 1: Predicted variation of the Mg VII $\lambda 280.75/\lambda 278.39$ ratio with density, computed from the CHIANTI database.

One point to note is that care must be taken as to whether intensities are given in energy or photon units. For example, the ratio shown in Fig. 1 would have to be multiplied by 278.39/280.75=0.992 to obtain the ratio in photon units. If two lines are far apart in wavelength, for example the Fe X λ257.26/λ184.54 ratio, then this factor can be large. The routines described in this document assume that the data have been calibrated by EIS_PREP in units of erg cm^{-2} s^{-1} sr^{-1} Å^{-1} (this is the default option for EIS_PREP). Theoretical line ratios are computed with CHIANTI in energy units.

3 Software summary

The basic steps to deriving a density map are given in the table below. The `eis_auto_fit` routine is described in EIS Software Note #16, and the other routines are described in more detail below.

	Step	IDL code
1	Fit Gaussians to the ratio's numerator and denominator lines	`eis_auto_fit, windata1, fit1` `eis_auto_fit, windata2, fit2`
2	Obtain theoretical ratio variation from CHIANTI	`ratio=eis_chianti_dens_ratio()`
3	Derive density and column depth map	`dens=eis_density(fit1,fit2,ratio)`

4 Gaussian fitting and line blending

The first step in the process is to perform Gaussian fitting to the emission lines. The software described here assumes that the routine `eis_auto_fit` has been used.

Before proceeding to derive densities, it is very important that the user be aware of any blending issues that affect the lines to be used. Not only is blending with other species important, but also self blending. Consider for example the Fe XII line found at 186.88 Å in the EIS spectrum. This line actually consists of two Fe XII transitions with laboratory wavelengths of 186.85 and 186.89 Å. The general recommendation is to fit the observed feature with a single Gaussian. When we come to derive a density using this line (usually relative to Fe XII λ195.12) it is very important that the atomic model includes both of the λ186.85 and λ186.89 transitions, otherwise the derived densities will be in error.

For inexperienced users it is best to either consult existing EIS papers that use density diagnostic lines, or contact someone on the EIS team if you are unsure of blending issues for your lines. It is also useful to be aware of typical densities for coronal structures: quiet Sun is typically about $10^{8.5}$ cm^{-3}, coronal holes about $10^{8-8.5}$ cm^{-3}; the outer regions of active regions about 10^{9} cm^{-3}; active region cores about $10^{9.5-10}$ cm^{-3}. Densities above 10^{10} cm^{-3} will mostly be caused by flaring. Therefore if you are analyzing a quiet Sun data-set but are finding densities of $10^{9.5}$ cm^{-3}, say, then it may indicate that blending is a problem for your lines.

5 Available software

After performing Gaussian fits to the two emission lines of the density diagnostic, there are two steps for deriving a density array. The first is to extract the theoretical variation of the line ratio with density, and this is performed with the routine `eis_chianti_dens_ratio.pro`:

```
IDL> ratio=eis_chianti_dens_ratio()
```

You will be asked to choose a particular line ratio from a menu. If your ratio is not listed it will be necessary to specify the precise transitions involved in the ratio - see Appendix A.

The theoretical ratio is automatically calculated using the latest version of the CHIANTI atomic database present in the user's Solarsoft distribution. The output `ratio` is an IDL structure containing the ratio value as a function of density as well as related information.

Once `ratio` has been created the two Gaussian fit structures can be used to create a density map. The call is:

```
IDL> dens=eis_density(fit1, fit2, ratio)
```

where `fit1` is the fit structure for the denominator line and `fit2` is the fit structure for the numerator line. The choice of the denominator and numerator line needs to match the choice made with the routine `eis_chianti_dens_ratio`. If the fit structures contain more than one line (i.e., they are the output from a multi-Gaussian fit), then it is necessary to specify which line to use. E.g.,

```
IDL> dens=eis_density(fit1, fit2, ratio, line1=0, line2=1)
```

which specifies the line with index 0 is to be used for the denominator line, and the line with index 1 as the numerator line.

An important point to note is that the density sensitive line pair will often have a significant wavelength separation, which means there will be a spatial offset in the Y-direction between the images formed from the two lines. (The offset is due to a tilt of the grating relative to the detector.) The offset, although usually small, can have a significant impact on the derived densities (Young et al., 2009). The routine `eis_density.pro` *automatically* corrects the intensity arrays for the Y-offset.

The output `dens` is an IDL structure with the following tags:

LDENS Logarithm (to base 10) of the electron number density (units: cm^{-3}).

DENS The electron number density (units: cm^{-3}).

LO Logarithm (to base 10) of the lower limit to the electron number density (units: cm^{-3}), based on the 1-σ error bars on the input line intensities.

HI Logarithm (to base 10) of the upper limit to the electron number density (units: cm^{-3}), based on the 1-σ error bars on the input line intensities.

RATIO The intensity ratio of the two density sensitive lines.

COLDEPTH The column depth (units: cm) of the emitting plasma.

COLDEPTH_ERR The error on the column depth (units: cm).

ARCSEC Scalar to be used for converting cm to arcsec (for column depth).

MISING The numerical value for missing pixels in the data arrays.

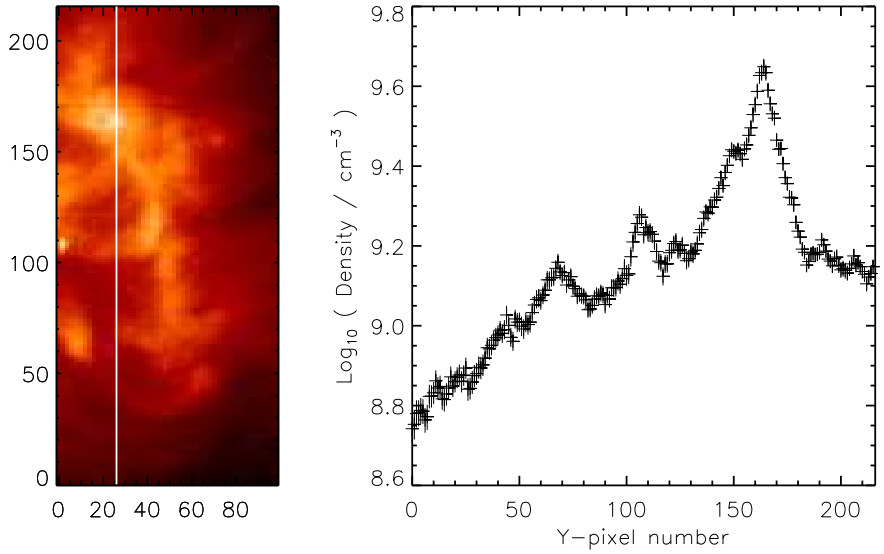

Figure 2: The left panel shows a Fe XIII (λ203.79+λ203.83)/λ202.04 density map from an EIS raster obtained on 2007 January 20 22:32 UT. The right panel shows a slice through the density map at X-pixel=13. Each density measurement is shown by a cross for which the vertical axis denotes the uncertainty of the measurement.

Figure 2 shows a density map derived from a raster from 2007 January 20 22:32 UT using the Fe XIII (λ203.79+λ203.83)/λ202.04 ratio, together with a slice through the data at X-pixel=13. The error bars on the density come from the 1σ error bars on the line intensities computed by `eis_auto_fit`. A density slice such as this can be plotted using

```
IDL> eis_dens_slice, dens, xpix=13
```

6 Column depths

The routine `eis_density` automatically creates an array of column depths (stored in the tag `coldepth`) in addition to densities. This section explains how the column depth is computed. There are two methods available: the emission measure (EM) method and the isothermal method. The default is the EM method. To see how the methods work, consider the expression for relating an observed line intensity, I, to atomic and physical parameters:

$$4\pi I = 0.83 \int G(T, N_{\rm e}) N_{\rm e}^2 {\rm d}h \qquad (1)$$

where $G(T, N_e)$ is the contribution function which contains atomic parameters for the emission line (see the CHIANTI user guide for more details about this function), N_e is the electron number density, and h is the column depth.

6.1 Emission measure method

This method is based on that of Pottasch (1963). The contribution function is generally a sharply peaked function with a width (in log T) of about 0.30 dex. The Pottasch method approximates the shape of G by a step function such that:

$$G(T) = \begin{cases} G_0 & \log T_{\max} - 0.15 \leq \log T \leq \log T_{\max} + 0.15 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The expression for the line intensity is then written as

$$4\pi I = 0.83\, G_0 \int N_e^2 \mathrm{d}h \tag{3}$$

The quantity $\int N_e^2 \mathrm{d}h$ is the emission measure that applies at the $T_{\max}$ of the ion.

We now assume that the density is constant in the region where the ion is formed (note that when we derived the density in the first place we made this assumption). We thus have

$$4\pi I = 0.83 G_0 N_e^2 h \tag{4}$$

i.e., a direct relation between the observed line intensity, the electron density and the column depth. The quantity G_0 contains purely atomic parameters and `eis_density` calls the CHIANTI routine `integral_calc` to compute it.

h can be computed with either the numerator or the denominator line, or both lines together (it can be seen that Eq. 4 holds for a linear combination of emission line intensities), and in fact the latter option is implemented in the `eis_density` code. This has an advantage as the contribution function for the sum of the numerator and denominator lines generally has a small density sensitivity, and so the G_0 value is weakly dependent on density. In the `eis_density` code, the G_0 values for each spatial pixel are computed for the densities that apply at those pixels.

6.2 Isothermal method

This method assumes that the plasma emitting the density diagnostic lines is an isothermal plasma and that the temperature is the temperature of maximum ionization of the plasma. This approximation actually yields a lower limit to the column depth. Since the density is also assumed constant then the expression for the line intensity becomes

$$4\pi I = 0.83 G(T_{\max}, N_e) N_e^2 h \tag{5}$$

and so the line intensity is again directly related to the electron density and column depth.

As for the EM method, h is computed by `eis_density` by summing the numerator and denominator intensities, and summing the G values.

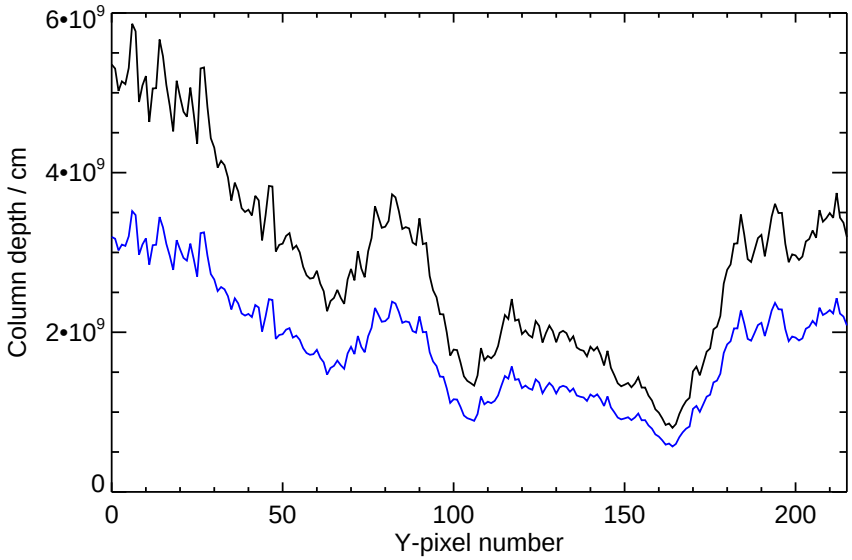

Figure 3: Column depths deduced from the Fe XIII (λ203.79+λ203.83)/λ202.04 ratio of the 2007 January 20 22:32 UT data-set are shown. The values represented by the black line have been computed with the EM method; the blue line shows values computed with the isothermal method.

6.3 Method comparison

Using the example data-set from 2007 January 20 22:32 UT (see also Figure 2), the column depths computed from the EM and isothermal methods are plotted in Figure 3. This demonstrates that the isothermal column depth is smaller than the EM column depth. The ratio varies between 1.4 and 1.7 across the plot.

Converting to arcseconds, the smallest and largest column depths from the EM method are 10 and 80 arcseconds, respectively.

6.4 Column depth uncertainty

If atomic data uncertainties are ignored, then the column depth uncertainty can be derived from the intensity and density uncertainties. The application of the density uncertainty is not straightforward, however, because the relationship between the theoretical intensity ratio variation and density is not linear. This means that the error on the measured intensity ratio does not translate to a standard error on the density. For example, if we consider the Fe XIII λ203.828/λ202.044 ratio, then using the atomic data from version 6 of CHIANTI shows that an observed ratio of 1.9 ± 0.3 corresponds to a density of $\log(N_{\rm e}/{\rm cm}^{-3}) = 9.65^{+0.14}_{-0.13}$, or $N_{\rm e} = 4.47^{+1.70}_{-1.16} \times 10^9$ cm^{-3}. The ratio in this case was chosen to lie in the center of the ratio sensitivity curve. Choosing a ratio close to the bottom of the sensitivity curve, 0.6 ± 0.3, gives more asymmetric errors for $\log N_{\rm e}$, although for $N_{\rm e}$ they become more symmetric: $\log(N_{\rm e}/{\rm cm}^{-3}) = 8.99^{+0.19}_{-0.30}$, or $N_{\rm e} = 9.77^{+5.33}_{-4.87} \times 10^8$ cm^{-3}. For ratios at the top of the sensitivity curve, e.g., 2.7 ± 0.3, the $N_{\rm e}$ errors become strongly asymmetric, but the $\log N_{\rm e}$ errors less so: $\log(N_{\rm e}/{\rm cm}^{-3}) = 10.08^{+0.27}_{-0.19}$, and $N_{\rm e} = 1.20^{+1.04}_{-0.42} \times 10^{10}$ cm^{-3}.

Generally density diagnostics will be applied when the observed ratios are close to the region

of maximum sensitivity of the ratio. Since the log N_e errors are closer to being symmetric in this case, then we will construct a standard error by simply averaging the upper and lower limits on log N_e. For example, for the Fe XIII ratio value of 1.9 ± 0.3, the standard error on log N_e is taken as ± 0.135. It is this error that is then propagated to determine the column depth uncertainity.

Considering the emission measure method for deriving the column depth (Eq. 4) and writing n for log N_e, and $k = 4\pi/0.83G_0$ we have

$$h = \frac{kI}{10^{2n}} = kIe^{an} \tag{6}$$

where $a = -2\ln 10 = -4.605$. Using standard propagation of error techniques (e.g., Sect. 3.2 of Bevington & Robinson, 2003), the uncertainty on h is given by

$$\sigma_h^2 = \sigma_I^2 \left(\frac{h}{I}\right)^2 + \sigma_n^2 a^2 h^2 \tag{7}$$

This is the formula that is implemented in `eis_density.pro`.

7 The filling factor

The filling factor of a plasma is the fraction of a volume that is contributing to the observed emission. Consider the case of a spherical volume that is resolved by the observing instrument such that the diameter is 10 pixels (=10″). Consider the spatial pixel at the center of the observed circle of emission. After performing the density calculation with `eis_density` you find a column depth of 5×10^8 cm at this pixel. This corresponds to about 7″. This then implies that only 7 of the 10″ path length through the diameter of the sphere is actually emitting the density diagnostic lines. Therefore the filling factor is 0.7.

The key point is that computing the filling factor *requires* that the user make some assumption about the emitting volume. Typically the assumption may be that the emitting volume is a sphere (for a bright point, for example), or a cylinder (for a loop).

References

Bevington, P. R., & Robinson, D. K. 2003, Data Reduction and Error Analysis for the Phyiscal Sciences, 3rd Ed., McGraw-Hill (New York)

Dere, K. P., & Mason, H. E. 1981, in Solar Active Regions, ed., F. Q. Orrall, (Boulder: Colorado Associated Univ. Press), 129

Dere, K. P., Landi, E., Mason, H. E., Monsignori-Fossi, B. C., & Young, P. R. 1997, A&AS, 125, 149

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Mariska, J. T. 1992, The Solar Transition Region, CUP

Pottasch, S. R. 1963, ApJ, 137, 945

Young, P. R., Watanabe, T., Hara, H., & Mariska, J. T. 2009, A&A, 495, 587

A How to specify a new line ratio in eis_chianti_density_ratio

The routine `eis_chianti_density_ratio` comes pre-loaded with a set of the most common EIS density diagnostics. If a diagnostic not on this list is needed, then the following procedure needs to be performed.

Consider the case of the Ni XVI λ194.02/λ185.23 ratio which lies in the EIS SW waveband, but is not listed by `eis_chianti_density_ratio`. For each atomic model within CHIANTI, a particular transition is identified by a unique index number. For the λ185.23 and λ194.02 these indices are 66 and 81, respectively. To specify these diagnostic lines to `eis_chianti_density_ratio`, the call is:

```
IDL> ratio=eis_chianti_dens_ratio('ni_16',66,81)
```

How are the indices obtained? This is done with the following commands:

```
IDL> em=emiss_calc(28,16)     ;  28 - Ni, 16 - XVI
IDL> em1=emiss_select(em,wra=[185,186],sel=sel)
```

A widget will appear asking you to select a line in the specified wavelength range. The line '185.230' is the correct one. The number contained in the output `sel` is the CHIANTI index of this line. Repeating this step for the 194.02 line gives the second index.

For blended lines an array of indices is input. E.g., [81,82].

B Document modification history

Version 2.3: Added Sect. 2.

Version 2.2: Added to the list of tags of the `dens` output (Sect. 5); modified Sect. 6.1 following implementation of density dependence of G_0 within `eis_density`.