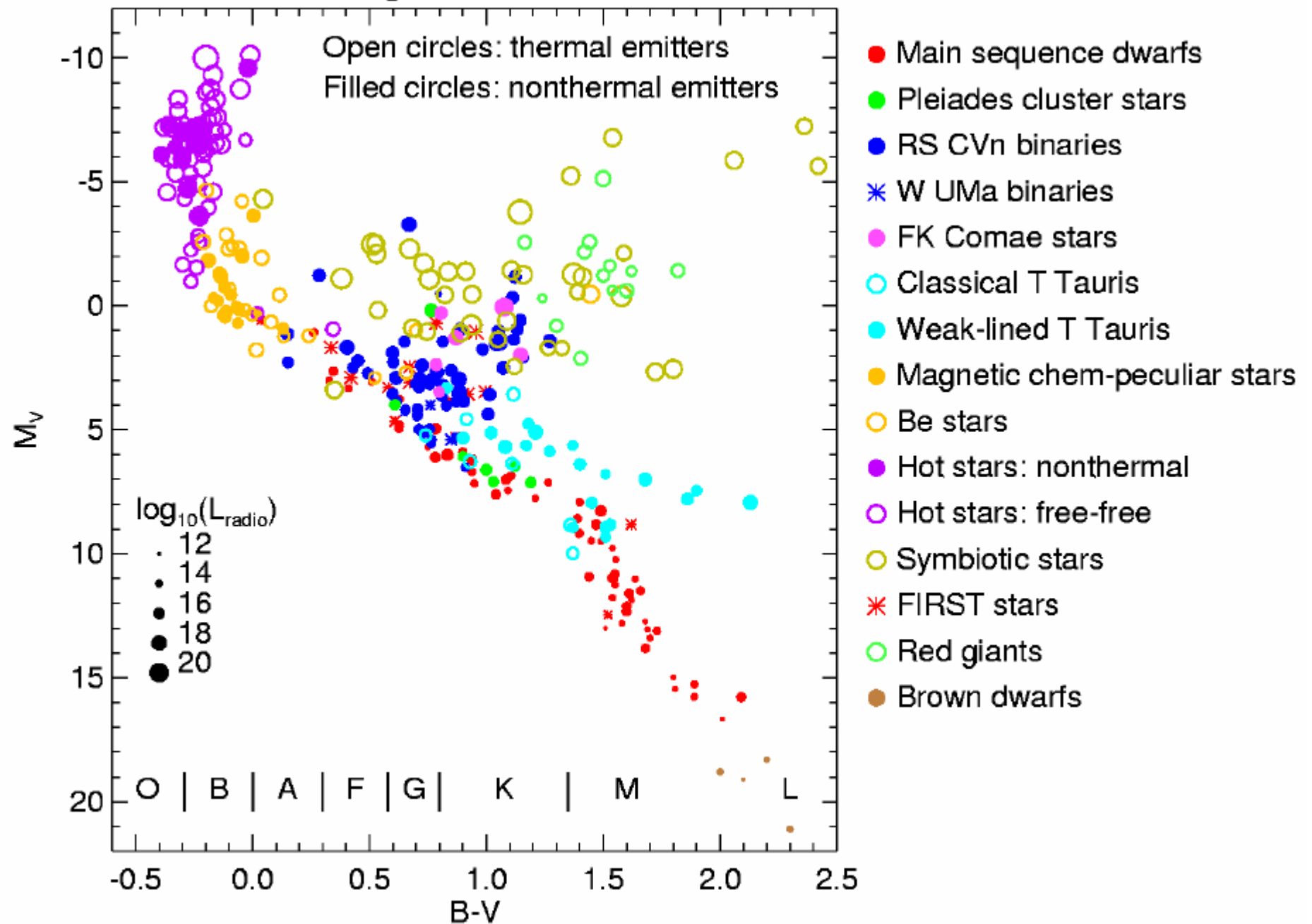


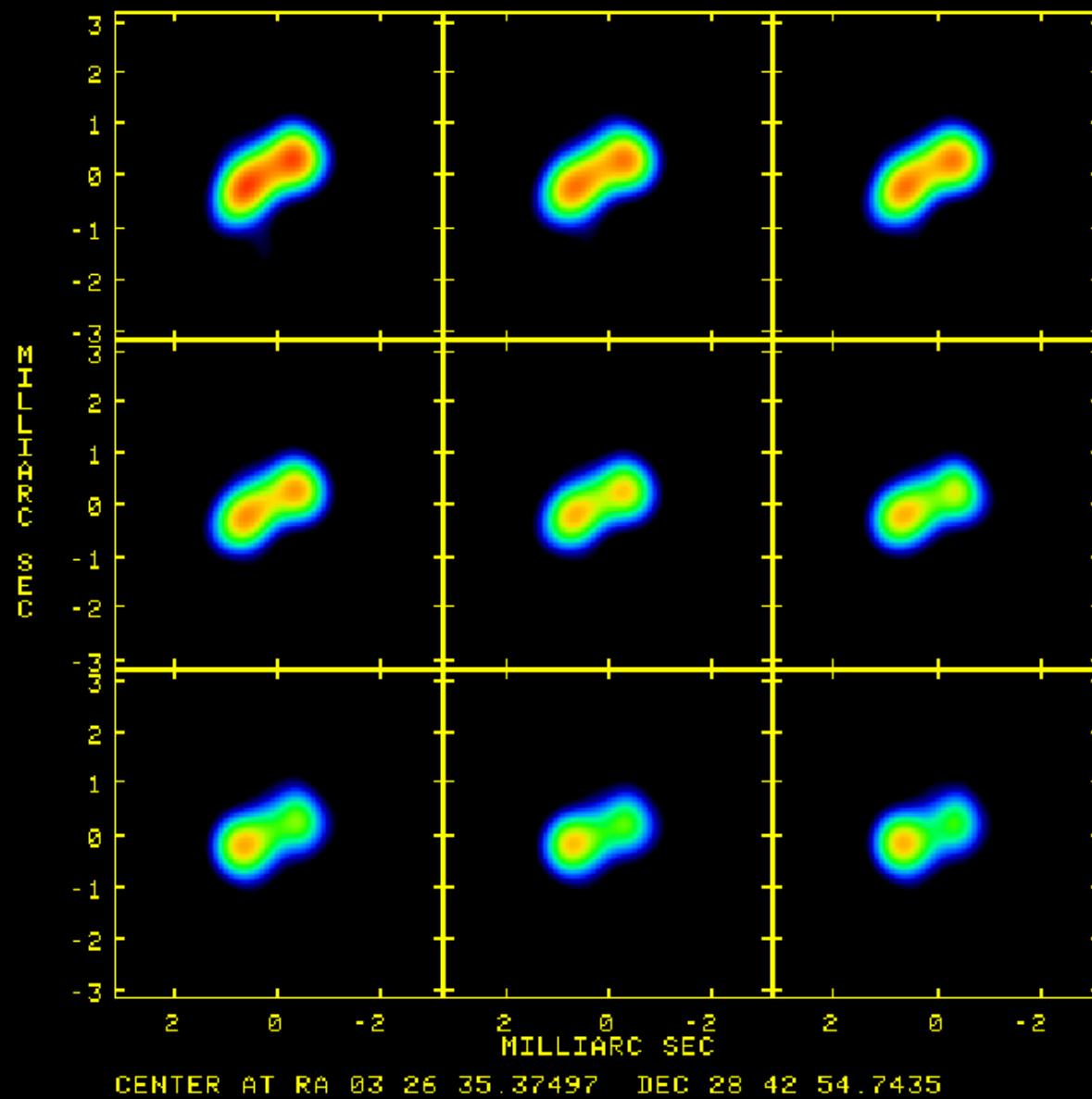
# Solar Radio Emission

T. S. Bastian

NRAO

# Radio H-R Diagram: Radio Luminosities





UX Ari

18 Nov 1995

$10^{35} - 10^{36}$  erg

VLBA

8.4 GHz

# Plan

- Preliminaries
- Emission mechanisms
  - Gyro-emission
    - thermal gyroresonance
    - nonthermal gyrosynchrotron
  - Thermal free-free emission
  - Plasma emission
- Solar radio phenomenology

# Preliminaries

Not surprisingly, the emission and absorption of EM waves is closely related to the **natural frequencies** of the material with which they interact.

In the case of a plasma, we encountered three frequencies:

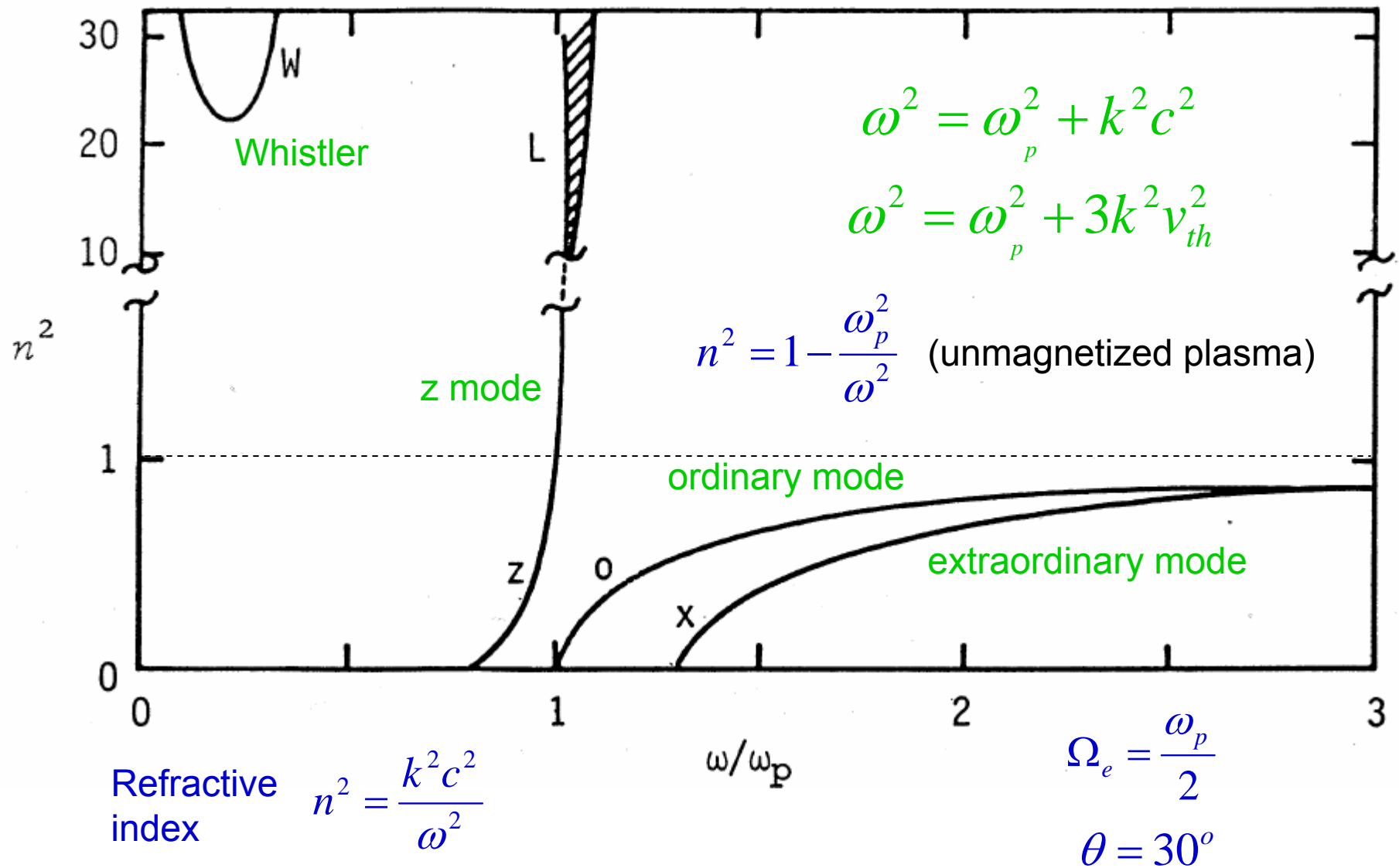
Electron plasma frequency  $\nu_{pe} \approx 9 n_e^{1/2}$  kHz

Electron gyrofrequency  $\nu_{Be} \approx 2.8 B$  MHz

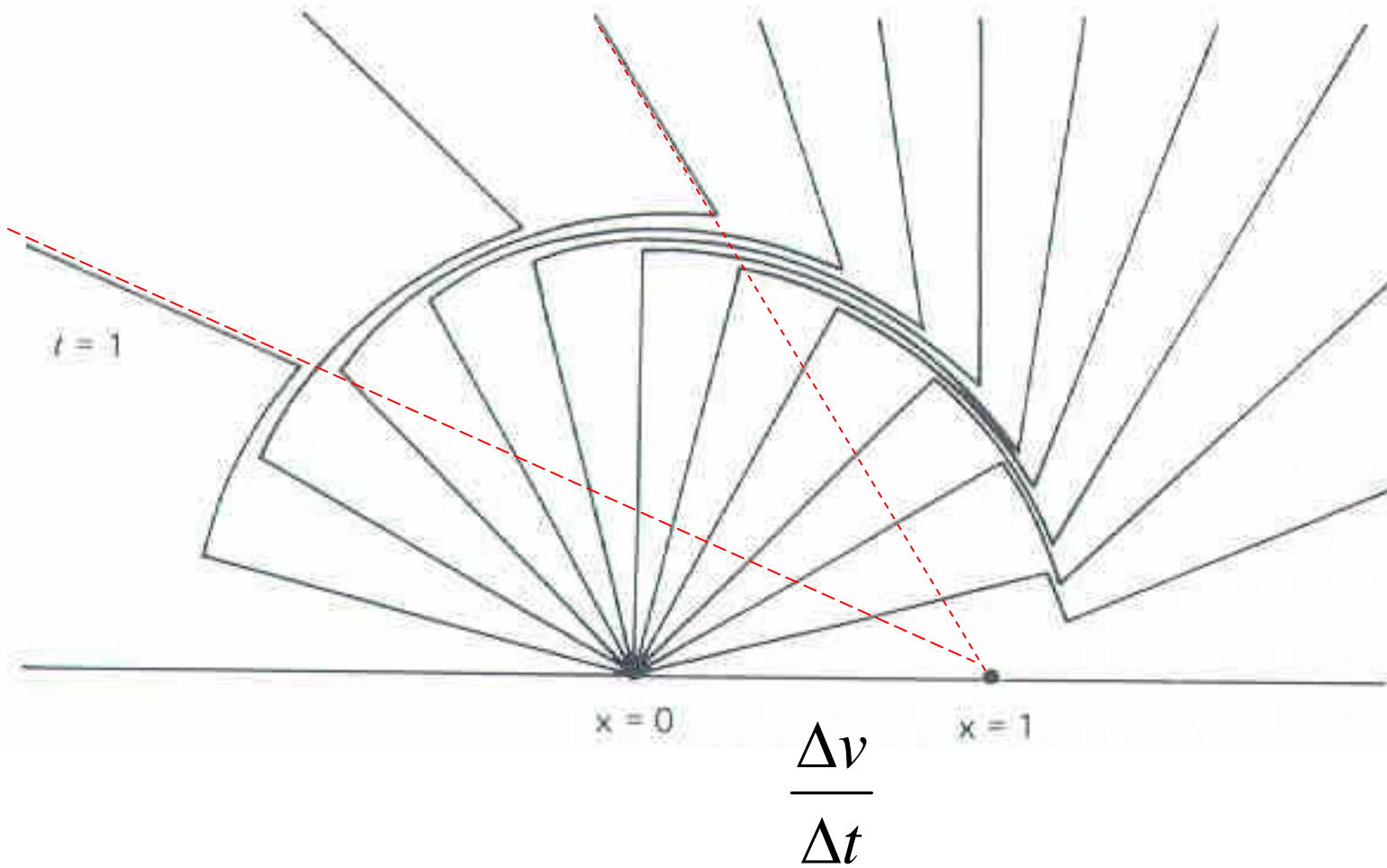
Electron collision frequency  $\nu_c \ll \nu_{pe}, \nu_{Be}$

These correspond to **plasma radiation**, **gyroemission**, and **“free-free”** or **bremsstrahlung** radiation.

# Wave modes supported by a cold, magnetized plasma



Charge with constant speed suddenly brought to a stop in time  $\Delta t$ .



The radiation power is given by the **Poynting Flux** (power per unit area: ergs cm<sup>-2</sup> s<sup>-1</sup> or watts m<sup>-2</sup>)

$$\mathbf{S} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{H} = \frac{c}{4\pi} E^2$$

Intuitive derivation  
will be posted with  
online version of  
notes.

$$|\mathbf{S}| = \frac{c}{4\pi} \left( \frac{q \ddot{x} \sin \theta}{rc^2} \right)^2 = \frac{q^2 \ddot{x}^2 \sin^2 \theta}{4\pi c^3 r^2}$$

$$P = \frac{2}{3} \frac{q^2 \ddot{x}^2}{c^3}$$

Power emitted into  
4π steradians

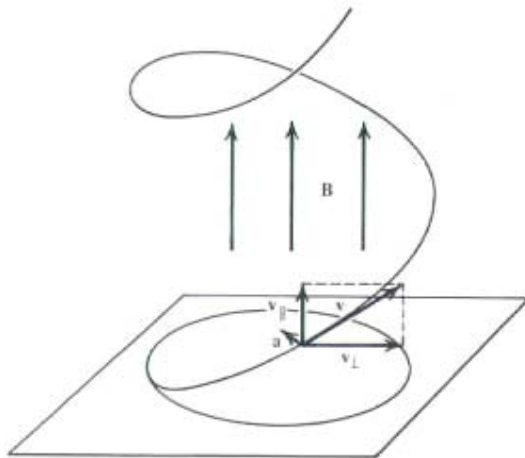
- Proportional to charge squared
- Proportional to acceleration squared
- Dipole radiation pattern along acceleration vector



# Gyroemission

Gyroemission is due to the acceleration experienced by an electron as it gyrates in a magnetic field due to the **Lorentz** force. The acceleration is **perpendicular** to the instantaneous velocity of the electron.

When the electron velocity is **nonrelativistic** ( $v \ll c$  or  $\gamma - 1 \ll 1$ ) the radiation pattern is just the dipole pattern.



Since the electron motion perpendicular to the magnetic field is circular, it experiences a constant acceleration perpendicular to its instantaneous velocity:

$$a_{\perp} = v_{Be} V_{\perp}$$

This can be substituted in to [Larmor's Eqn](#) to obtain

$$P = \frac{2e^2}{3c^3} v_{Be}^2 V_{perp}^2$$

In fact, this expression must be modified in when the electron speed is relativistic (i.e., [near c](#)):

$$P = \frac{2e^2}{3c^3} \gamma^4 v_{Be}^2 V_{perp}^2$$

$$v_{Be} = \frac{eB}{\gamma mc}$$

When the electron is relativistic ( $v \sim c$  or  $\gamma \gg 1$ ) we have, in the rest frame of the electron

$$\frac{dP'}{d\Omega'} = \frac{q^2 \omega^2}{4\pi c^3} \sin^2 \Theta'$$

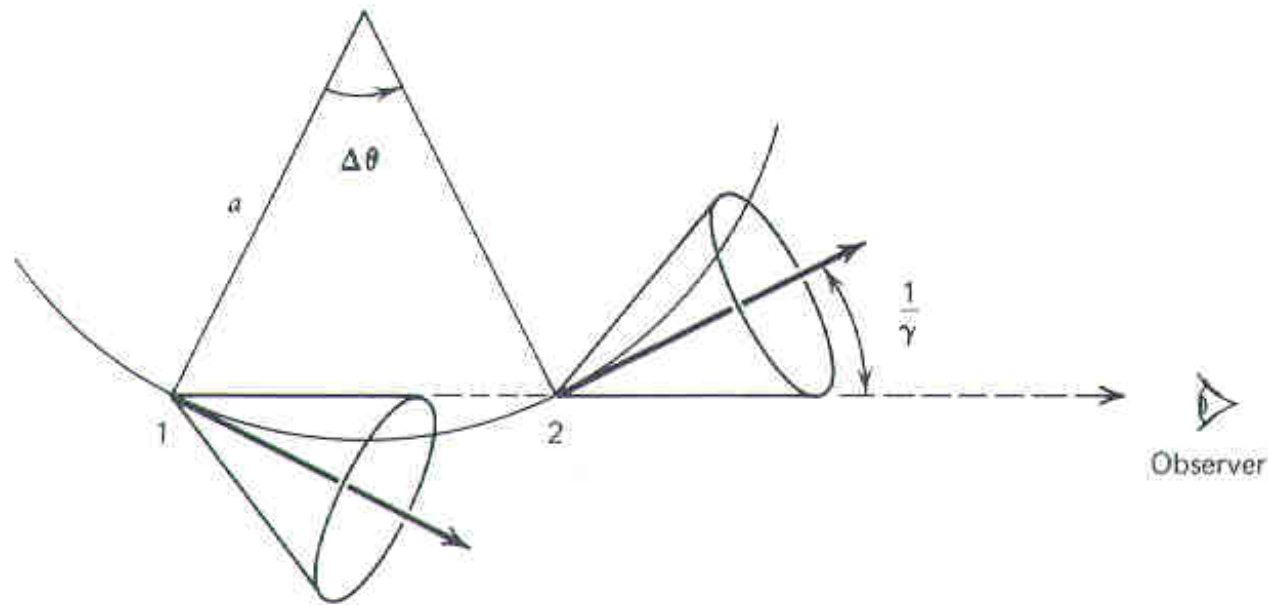
which, when transformed into the frame of the observer

$$\frac{dP}{d\Omega} = \frac{q^2 \omega^2}{4\pi c^3} \frac{1}{(1 - \beta \mu)^4} \left[ 1 - \frac{\sin^2 \theta \cos^2 \phi}{\gamma^2 (1 - \beta \mu)^2} \right]$$

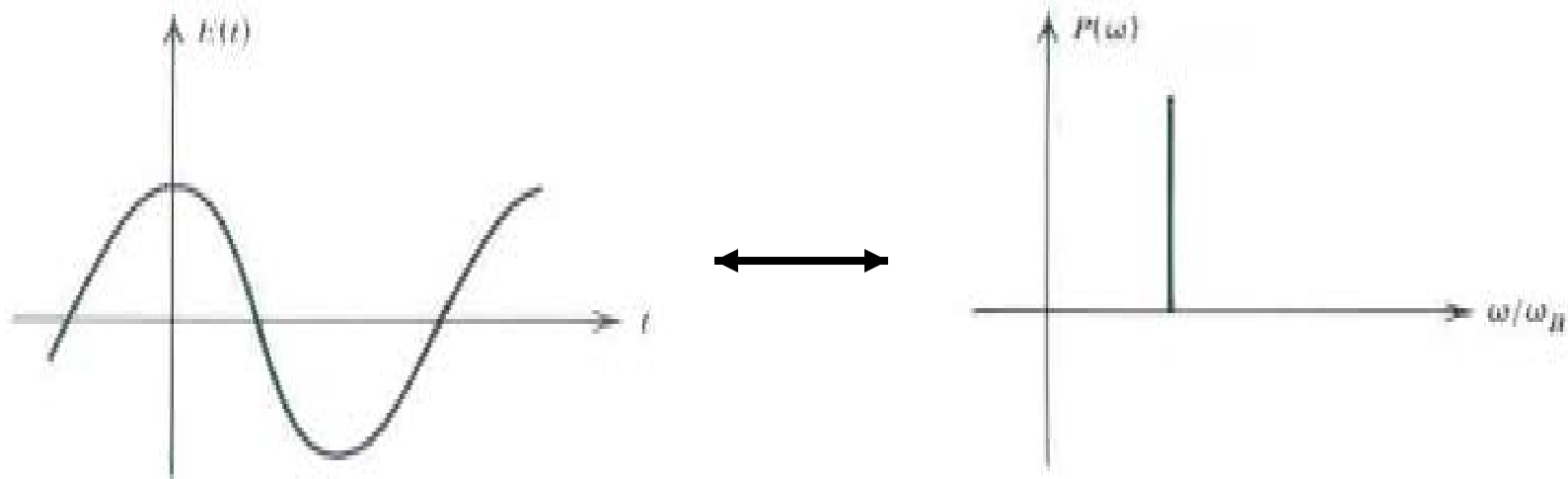


Strongly forward peaked!

What this means is that an observer sees a signal which becomes more and more sharply **pulsed** as the electron increases its speed (and therefore its energy).

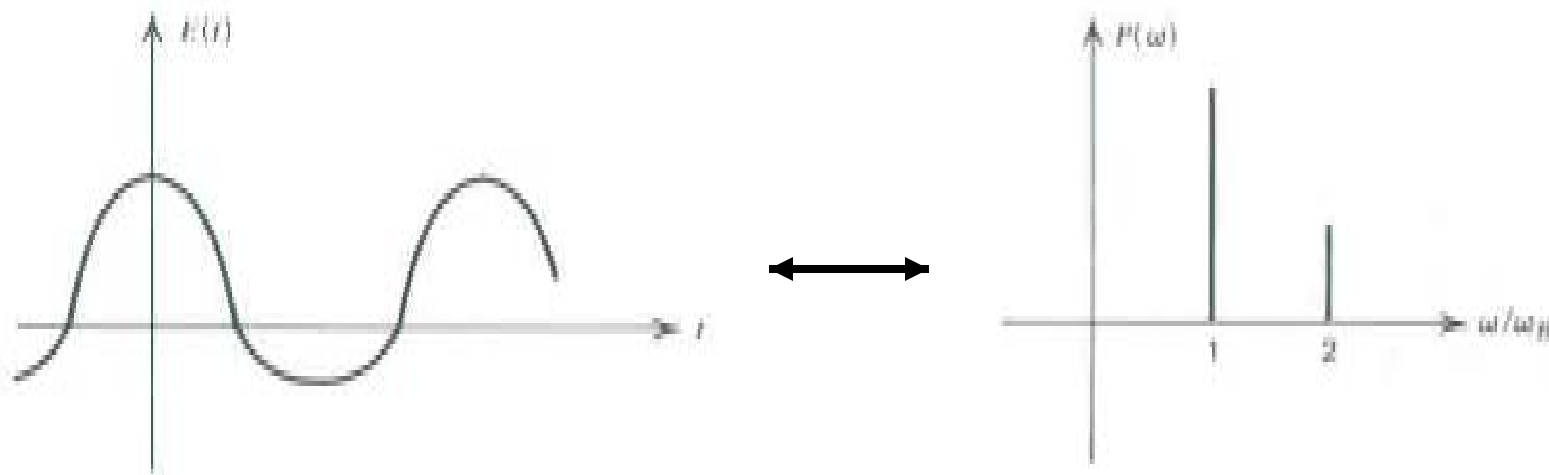


For a **nonrelativistic** electron, a **sinusoidally** varying electric field is seen which has a period  $2\pi/\Omega_{Be}$ ,



And the power spectrum yields a **single tone** (corresponding to the **electron gyrofrequency**).

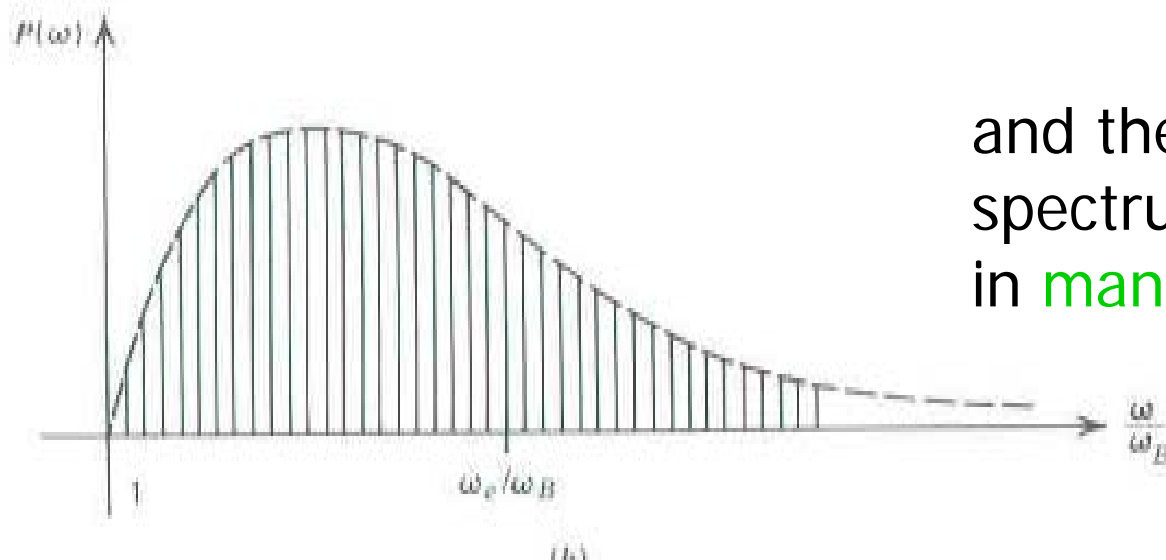
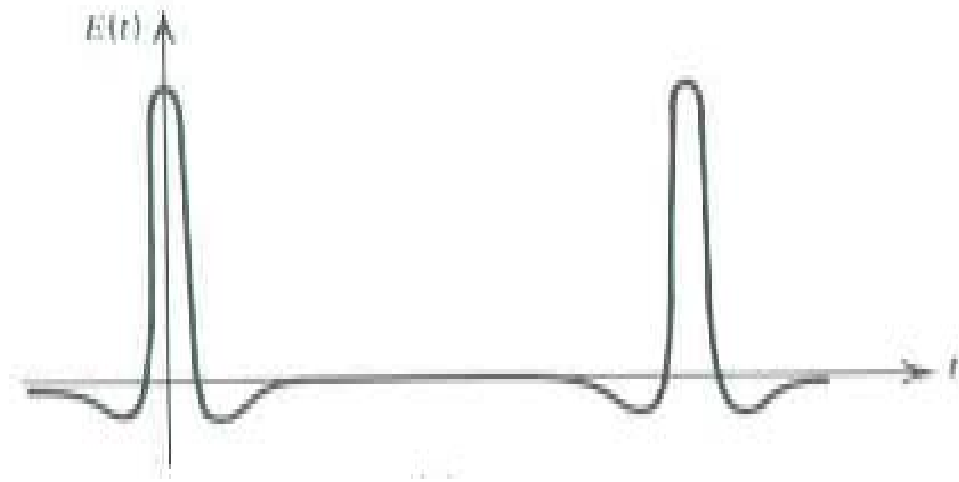
As the electron energy increases, **mild beaming** begins and the observed variation of the electric field with time becomes **non-sinusoidal**.



The power spectrum shows power in **low harmonics** (integer multiples) of the **electron gyrofrequency**.

Gyroemission at low harmonics of the gyrofrequency is called **cyclotron radiation** or **gyroresonance emission**.

When the electron is **relativistic** the time variation of  $E$  is **highly non-sinusoidal**...



and the power spectrum shows power in **many harmonics**.

A detailed treatment of the spectral and angular characteristics of electron gyroemission requires a great deal of care.

A precise expression for the emission coefficient that is valid for all electron energies is not available. Instead, expressions are derived for various electron energy regimes:

Non-relativistic:  $\gamma - 1 \ll 1$  (thermal)

cyclotron or gyroresonance radiation

Mildly relativistic:  $\gamma - 1 \sim 1-5$  (thermal/non-thermal)

gyrosynchrotron radiation

Ultra-relativistic:  $\gamma - 1 \gg 1$  (non-thermal)

synchrotron radiation



Synchrotron radiation is encountered in a variety of sources. The electrons involved are generally **non-thermal** and can often be parameterized in terms of a **power law** energy distribution:

$$N(E)dE = CE^{-\delta}dE$$

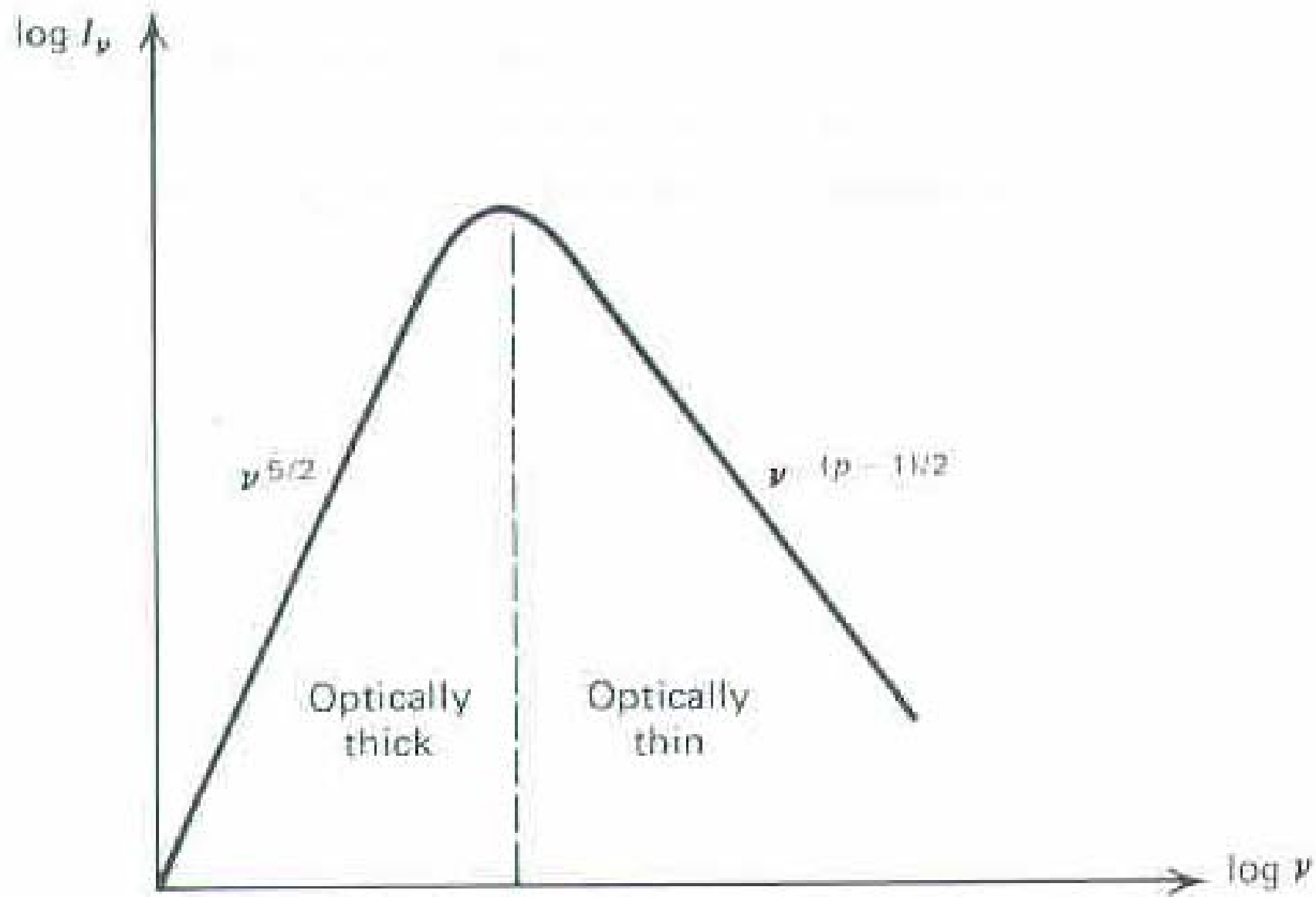
In this case, we have

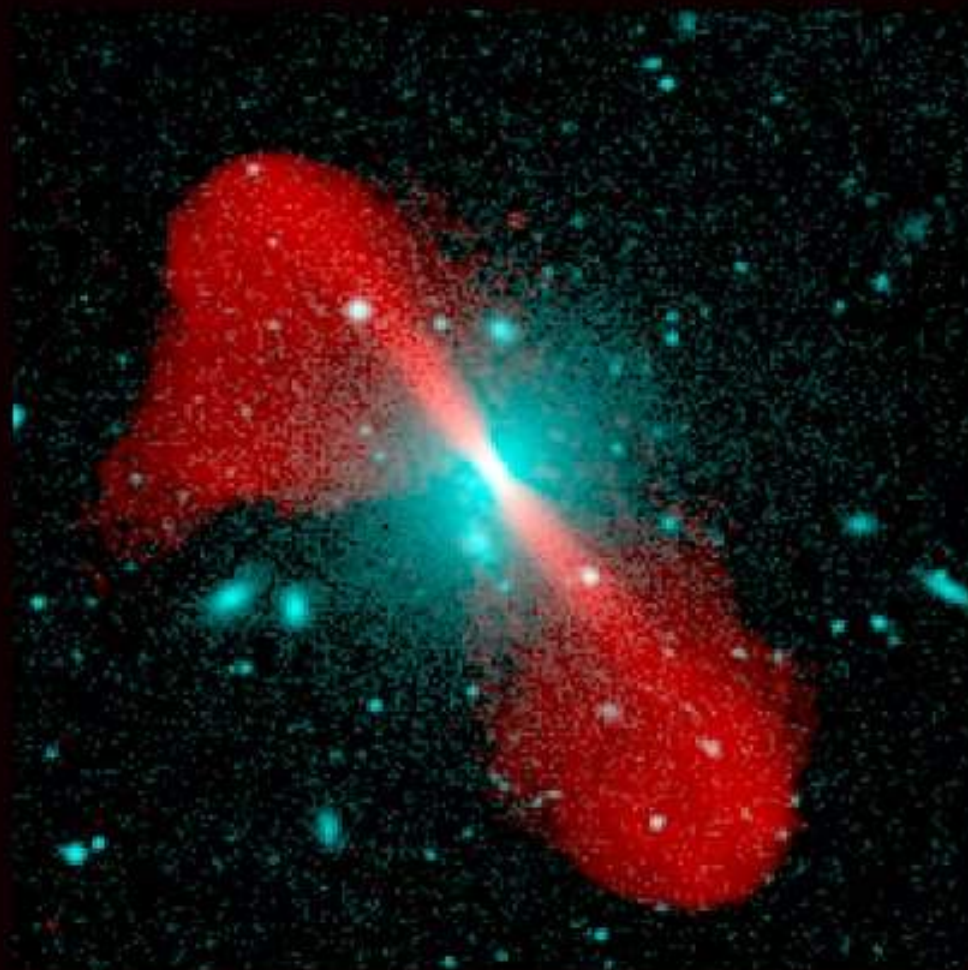
$$P(\nu) \propto \nu^{\frac{\delta-1}{2}} \quad \tau \ll 1$$

when the source is optically thin and

$$P(\nu) \propto \nu^{5/2} \quad \tau \gg 1$$

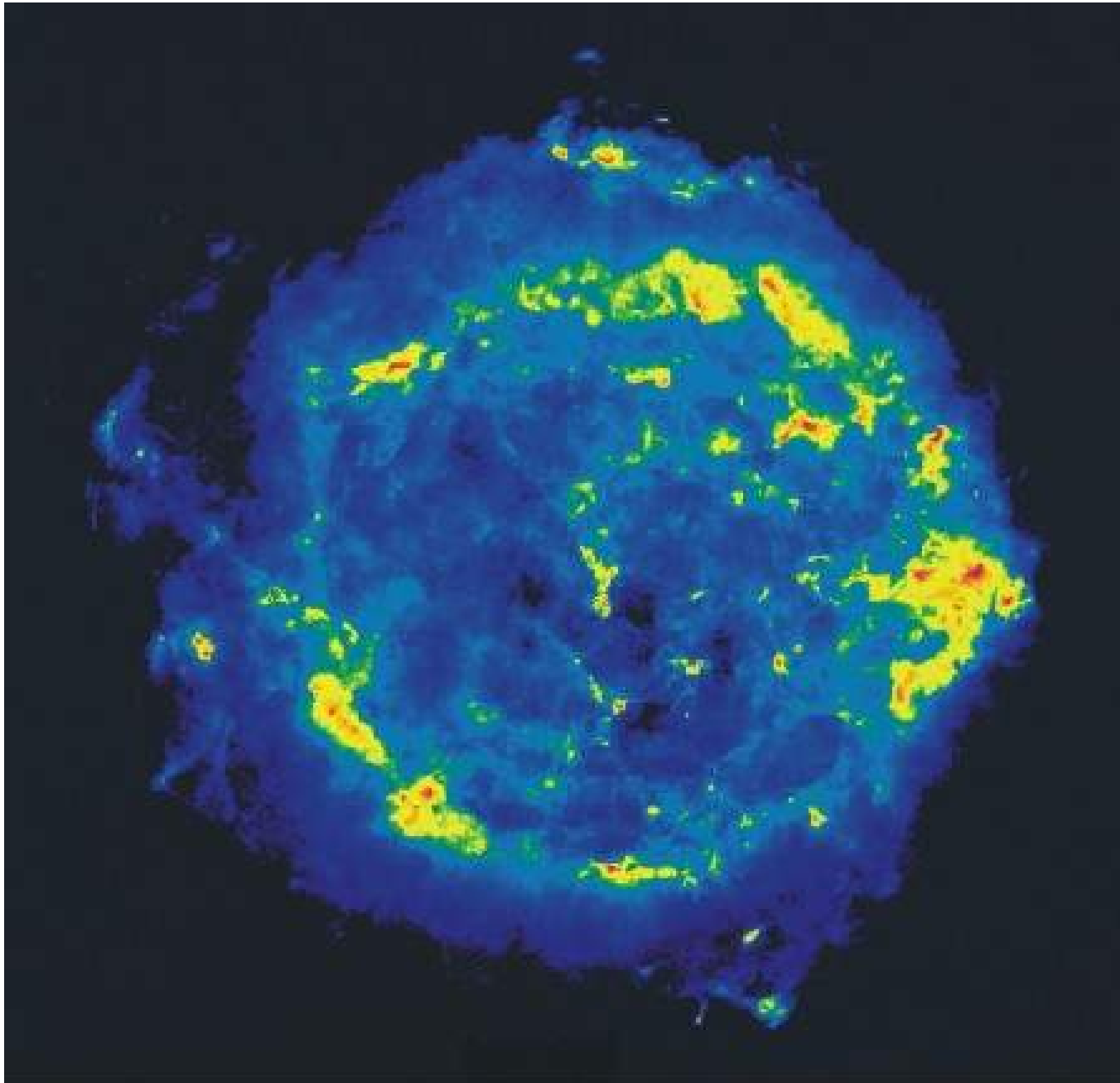
when the source is optically thick (or self absorbed).





Radio Galaxy 3C296  
Radio/optical superposition

Copyright (c) NRAO/AUI 1999



Observers: P.E. Argerhofer, R. Braun, S.F. Cull, R.A. Perley, R.J. Tutts

# Thermal gyroresonance radiation

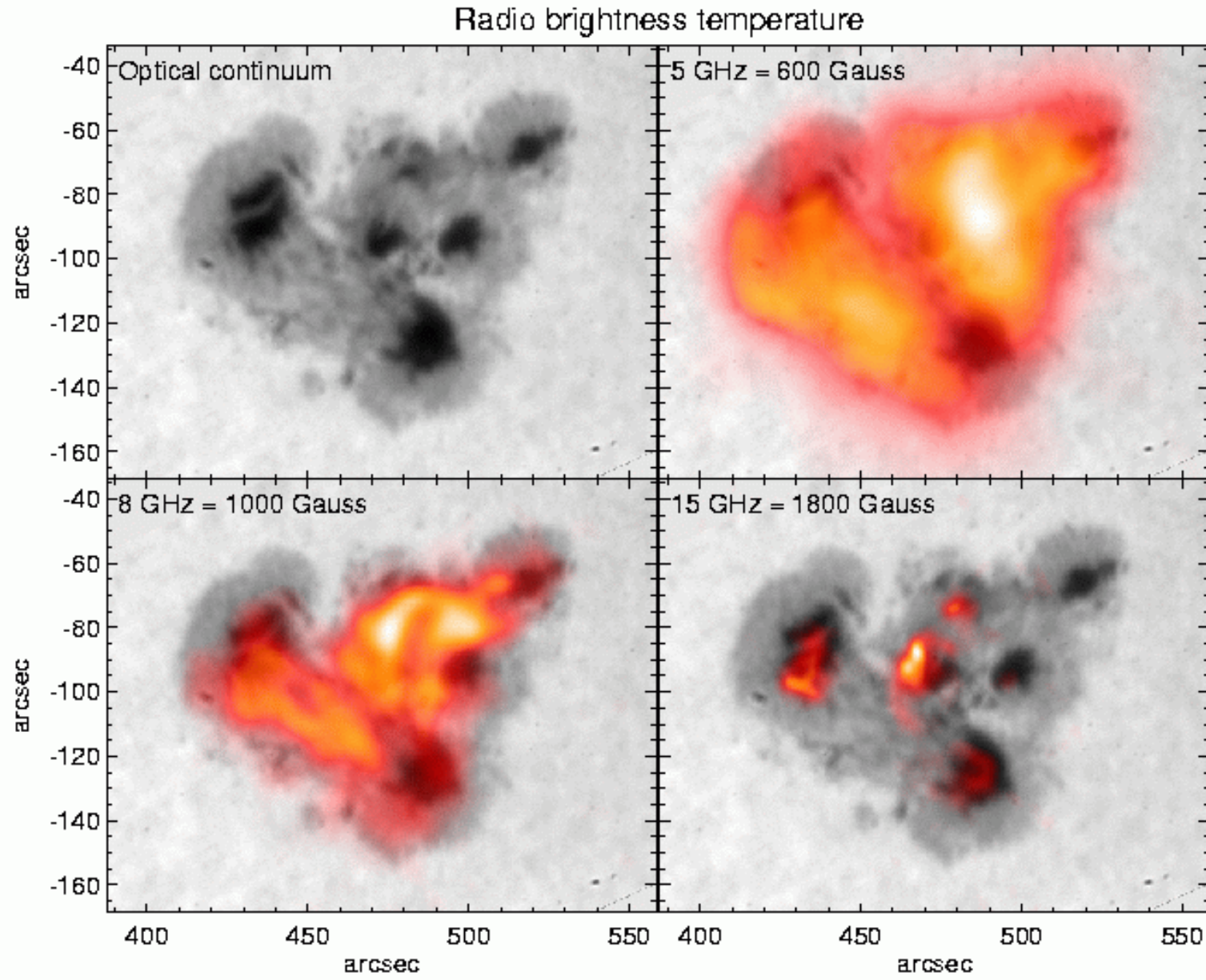
- Harmonics of the gyrofrequency

$$\nu = s \nu_B = s \frac{eB}{2\pi m_e c} = 2.8 \times 10^6 s B \text{ Hz}$$

- Two different modes, or circular polarizations  
( $\sigma=+1$  o-mode,  $\sigma=-1$  x-mode)

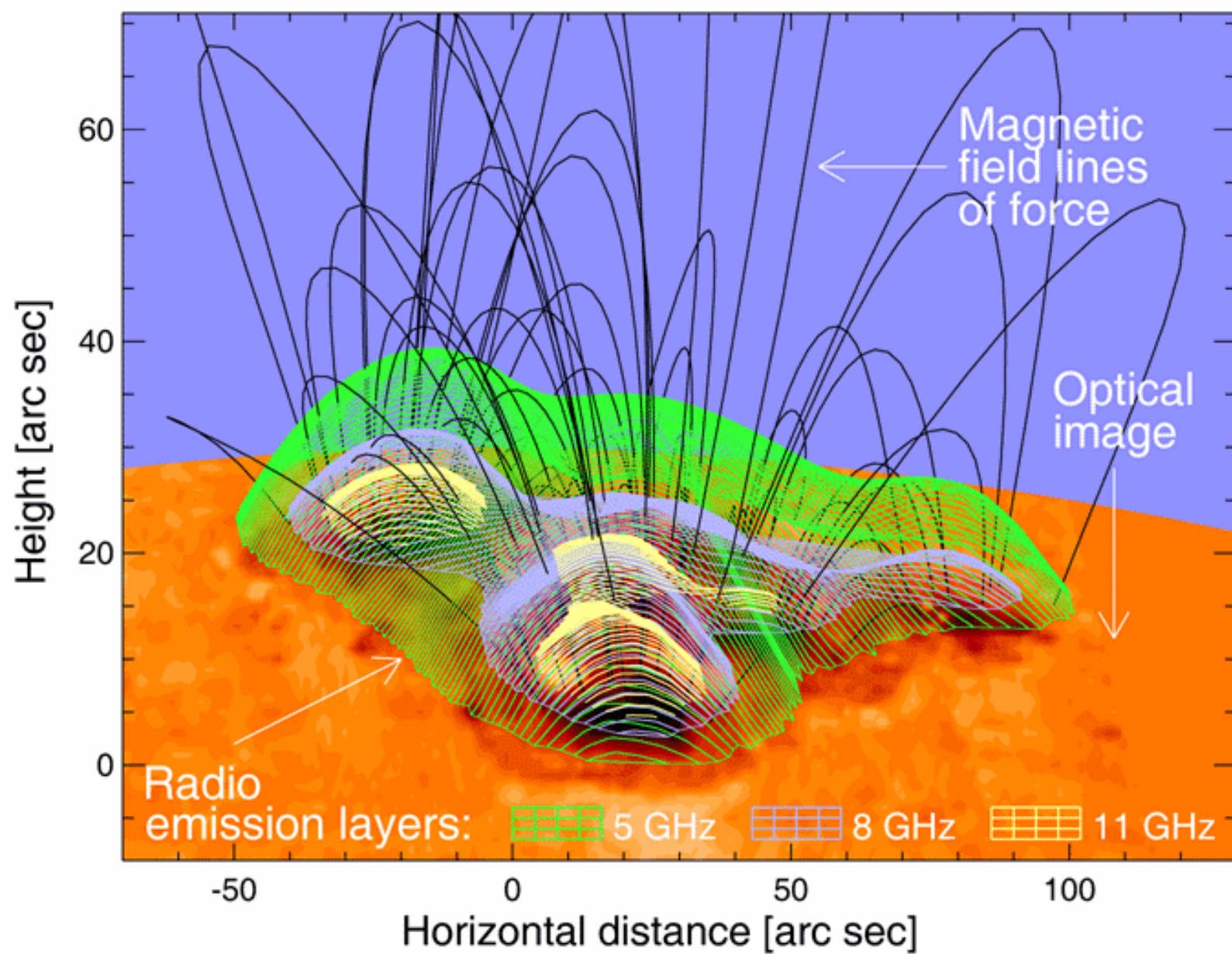
$$\alpha_{x,o} = \left(\frac{\pi}{2}\right)^{5/2} \frac{2}{c} \frac{\nu_p^2}{\nu} \frac{s^2}{s!} \left(\frac{s^2 \beta_0^2 \sin^2 \theta}{2}\right)^{s-1} (1 - \sigma \cos \theta)^2$$

- Typically,  $s = 2$  (o-mode),  $s = 3$  (x-mode)



from Lee et al (1998)

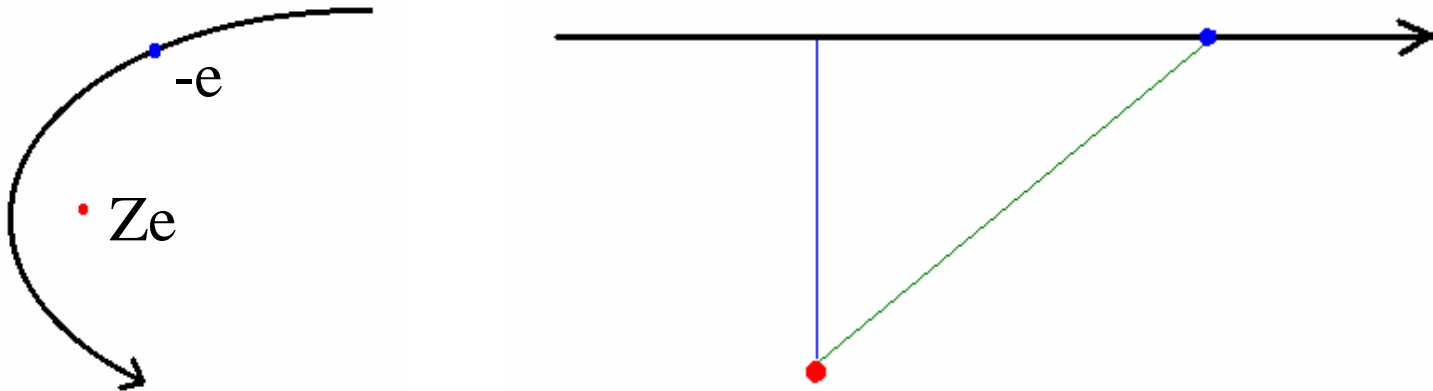




## Thermal free-free radiation

Now consider an electron's interaction with an ion. The electron is accelerated by the **Coulomb field** and therefore radiates electromagnetic radiation.

In fact, the electron-ion collision can be approximated by a **straight-line trajectory** with an **impact parameter  $b$** . The electron experiences an acceleration that is largely **perpendicular** to its straight-line trajectory.





For a thermal plasma characterized by temperature  $T$ , the absorption coefficient is

$$\alpha_{\nu}^{ff} = \frac{4\pi e^6}{3mhc} \left( \frac{2\pi}{3km} \right)^{1/2} T^{-1/2} Z^2 n_e n_i \nu^{-3} (1 - e^{-h\nu/kT}) \bar{g}^{ff}$$

In the Rayleigh-Jeans regime this simplifies to

$$\alpha_{\nu}^{ff} = \frac{4\pi e^6}{3mkc} \left( \frac{2\pi}{3km} \right)^{1/2} T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}^{ff}$$

$$\alpha_{\nu}^{ff} = 0.018 T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}^{ff}$$

where  $\bar{g}^{ff}$  is the (velocity averaged) Gaunt factor.

$$\alpha_{\nu}^{ff} \propto T^{-3/2} n_e^2 \nu^{-2}$$

$$\tau_{\nu}^{ff} = \alpha_{\nu}^{ff} L \propto T^{-3/2} (n_e^2 L) \nu^{-2}$$

The quantity  $n_e^2 L$  is called the **column emission measure**.

Notice that the optical depth  $\tau^{ff}$  decreases as the temperature  $T$  **increases** and/or the column emission measure **decreases** and/or the frequency  $\nu$  **increases**.

## A brief aside

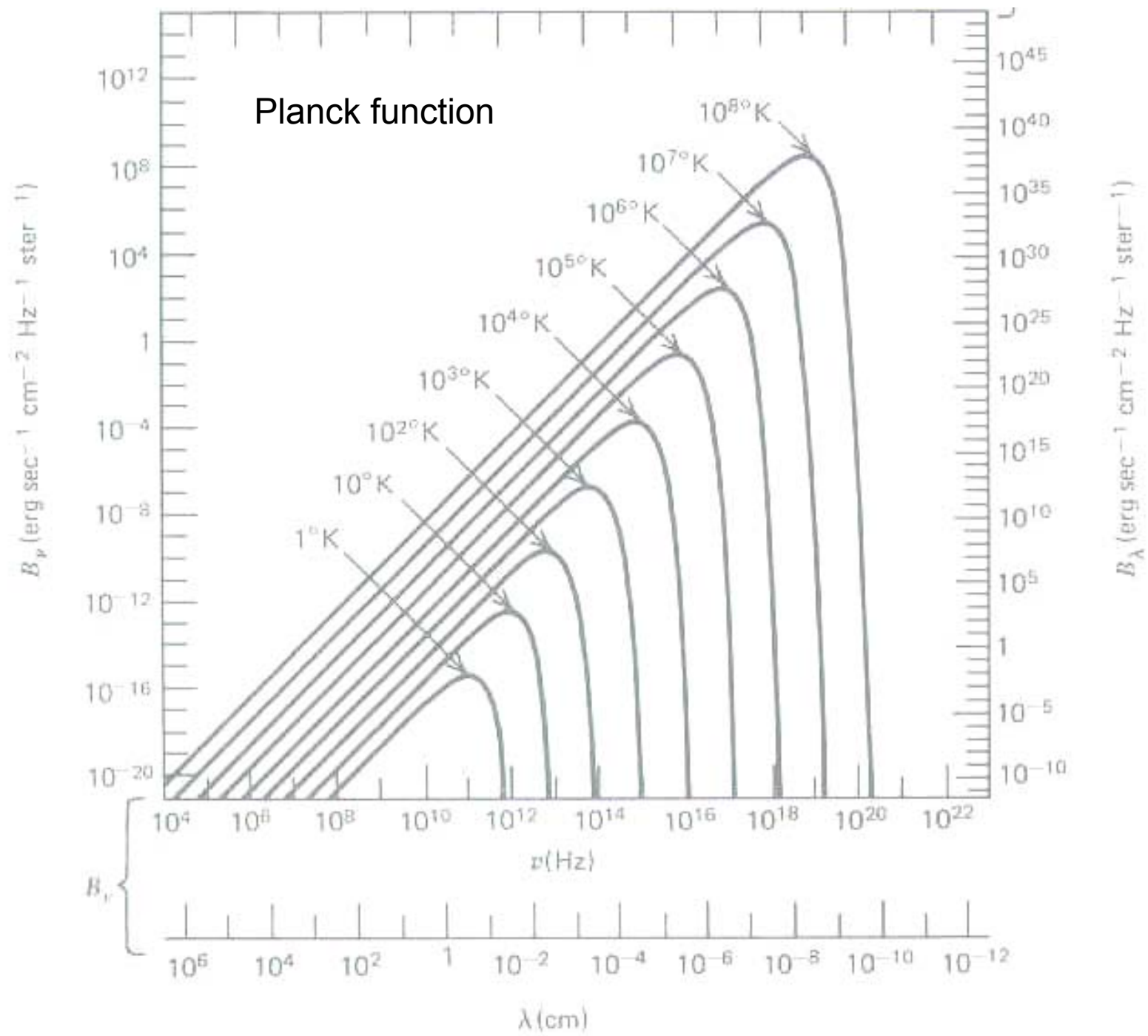
- Radio astronomers express flux density in units of Janskys

$$1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$$

- Solar radio physics tends to employ solar flux units (SFU)

$$1 \text{ SFU} = 10^4 \text{ Jy}$$

- While **specific intensity** can be expressed in units of **Jy/beam** or **SFU/beam**, a simple and intuitive alternative is **brightness temperature**, which has units of **Kelvin**.



Note that at radio wavelengths

$$h\nu / kT \ll 1 \rightarrow e^{h\nu/kT} - 1 \approx 1 + \frac{h\nu}{kT} - 1 = \frac{h\nu}{kT}$$

The Planckian then simplifies to the Rayleigh-Jeans Law.

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \approx \frac{2\nu^2}{c^2} kT$$

It is useful to now introduce the concept of brightness temperature  $T_B$ , which is defined by

$$I_\nu = B_\nu(T_B) = \frac{2\nu^2}{c^2} kT_B$$

Similarly, we can define an effective temperature  $T_{eff}$ :

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{2\nu^2}{c^2} kT_{eff}$$

The radiative transfer equation is written

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

which describes the change in specific intensity  $I_\nu$  along a ray. Emission and absorption are embodied in  $\alpha_\nu$  and  $j_\nu$ , respectively. The optical depth is defined through  $d\tau_\nu = \alpha_\nu ds$  and the RTE can be written

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Using our definitions of **brightness temperature** and **effective temperature**, the RTE can be recast as

$$\frac{dT_B}{d\tau_\nu} = -T_B + T_{eff}$$

For a homogeneous source it has the solution

$$T_B = T_{eff} (1 - e^{-\tau_\nu})$$

$$T_B = T_{eff} = T$$

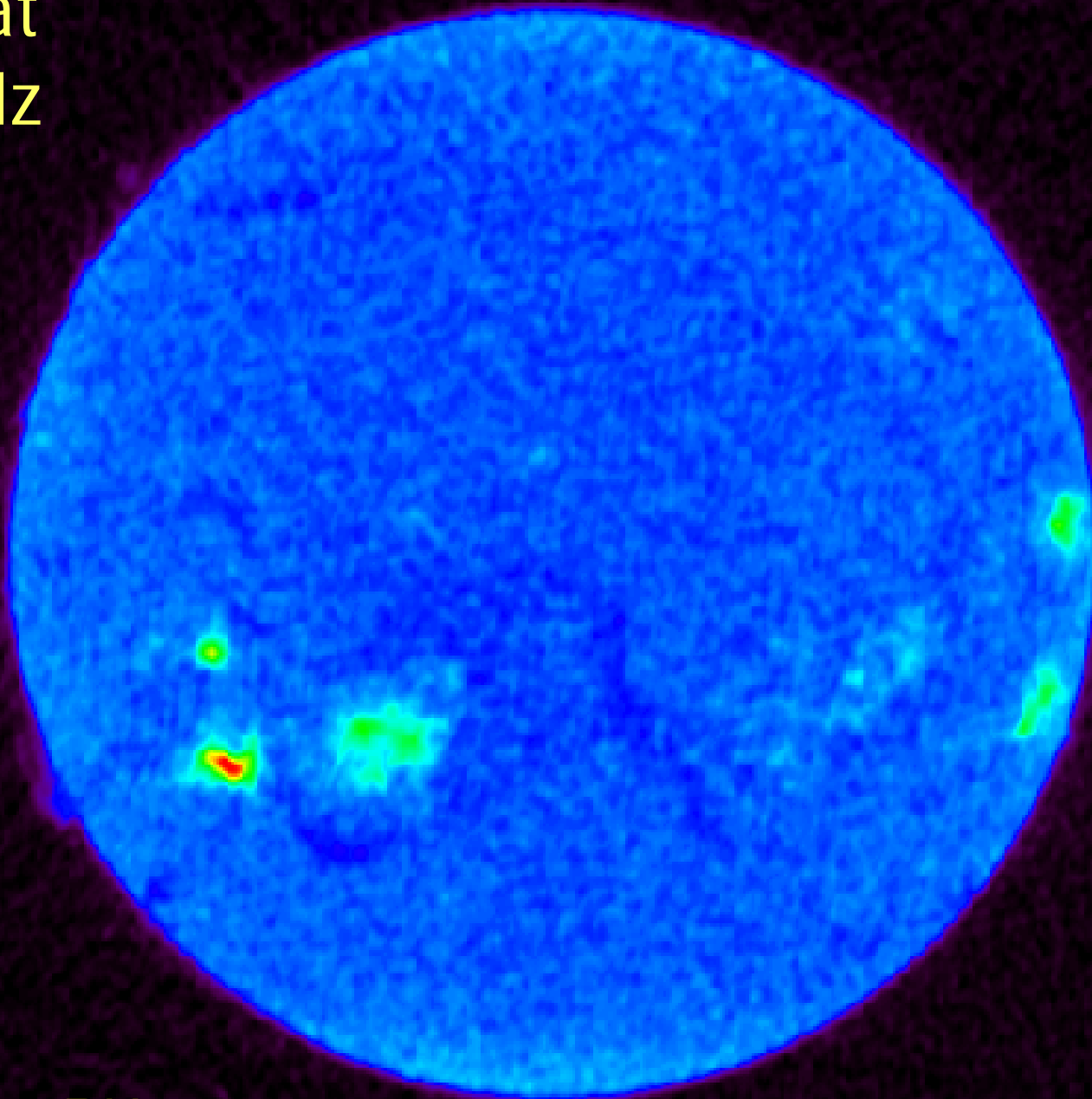
$$\tau_\nu \gg 1$$

$$T_B = T \tau_\nu^{ff} \propto T^{-1/2} \nu^{-2} n_e^2 L$$

$$\tau_\nu \ll 1$$

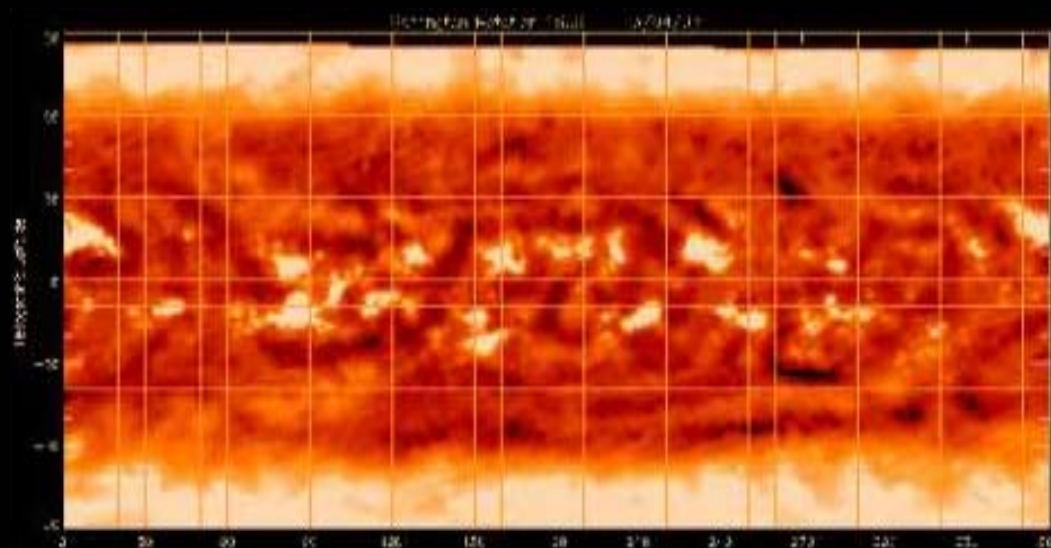
Brightness temperature is a simple and intuitive expression of specific intensity.

Sun at  
17 GHz

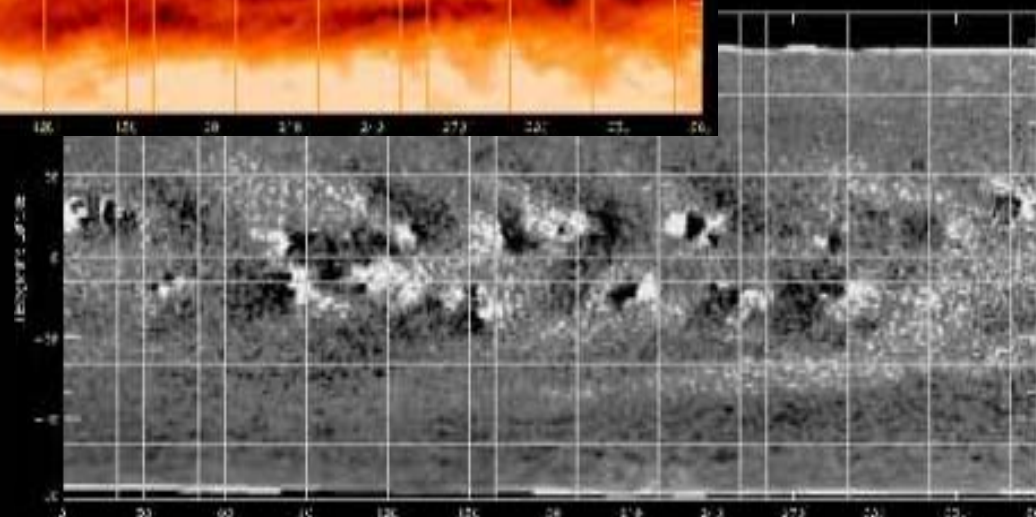


Nobeyama RH

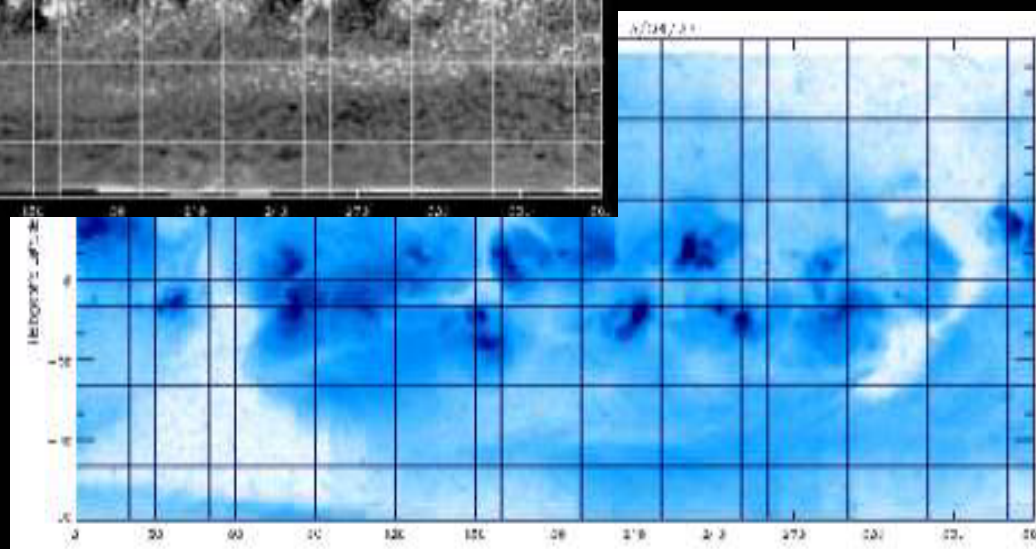




17 GHz



B gram



SXR

- A magnetic field renders a plasma “birefringent”
- The absorption coefficient for the two magnetoionic modes, x and o, is

$$\alpha_{x,o} = \left(\frac{2}{\pi}\right)^{1/2} \frac{1}{3c} \frac{\nu_p^2}{(\nu + \sigma \nu_B \cos \theta)^2} \frac{4\pi e^4 \sum_i Z_i^2 n_i}{m^{1/2} (kT)^{3/2}} \Lambda$$

- The x-mode has higher opacity, so becomes optically thick slightly higher in the chromosphere, while o-mode is optically thick slightly lower

- Degree of circular polarization

$$\rho_c = \frac{T_R - T_L}{T_R + T_L}$$

# Free-Free Opacity

$$\rho_C = \beta \frac{v_B}{v} \cos \theta \propto B_l$$

$$\beta = \frac{d \ln T}{d \ln v}$$

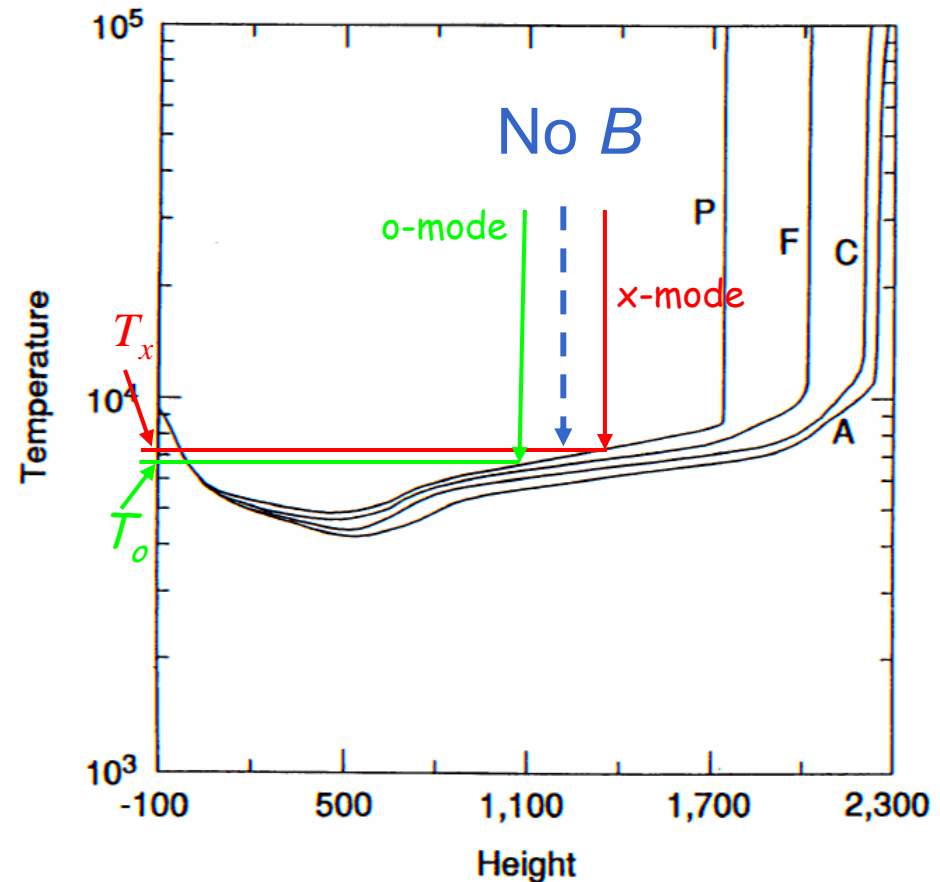
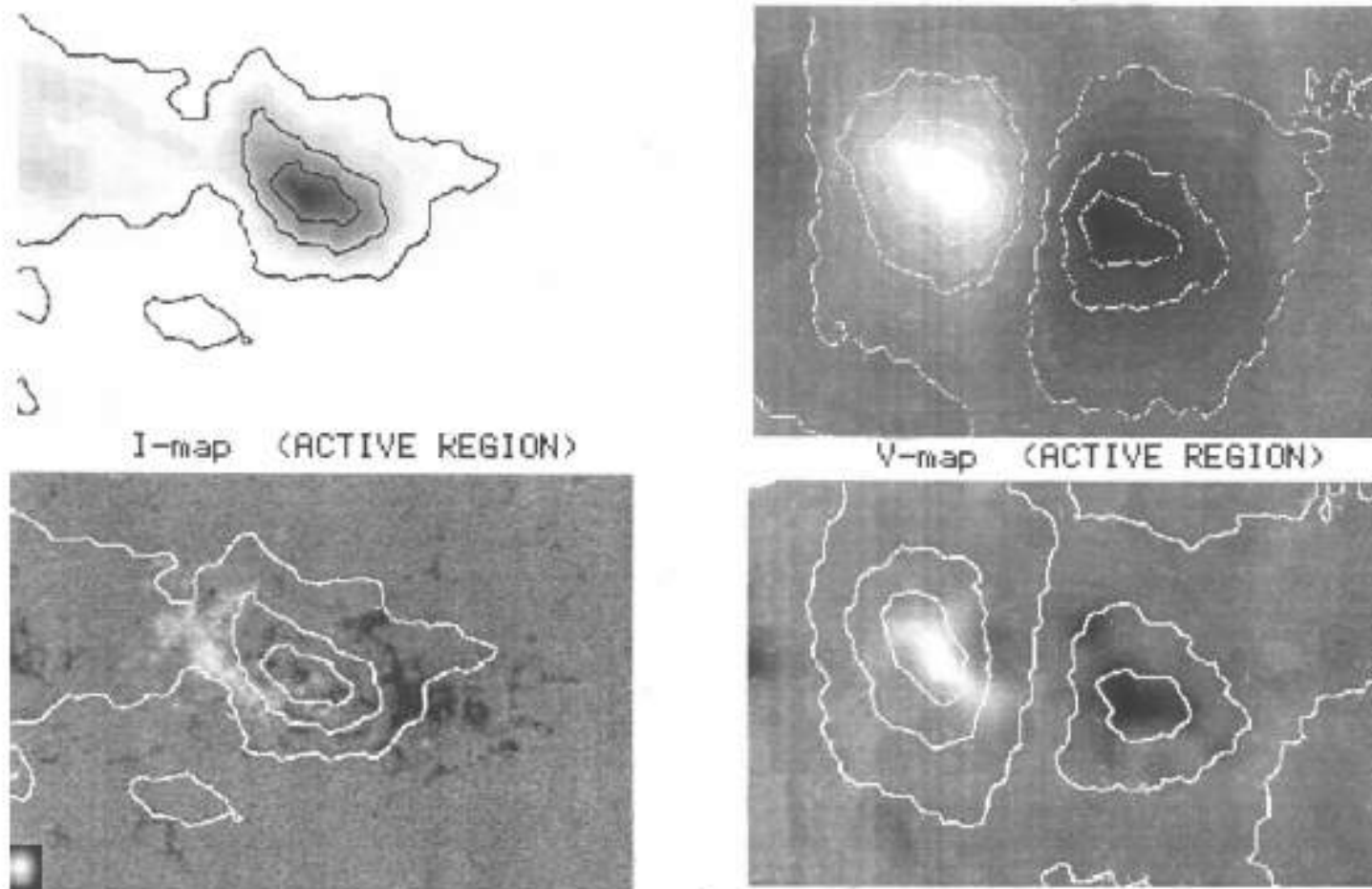


FIG. 3.—Temperature structure of our models A, C, F, and P. The height is measured in kilometers from the level; the temperature is in kelvins.



*Figure 5.1.* Radio maps of the AR observed on June 09, 1995 using Nobeyama radio heliograph at  $\lambda = 1.76\text{cm}$ . Contours present the brightness distribution. Maximum in I channel ( $T_b = 27 \cdot 10^3 K$ ). Maximum in V-channel  $T_b^V = 440K$ . Maximum degree of polarization  $P = 2.8\%$ . The region maps are overlapped by gray scale magnetograms. For V-maps they are averaged by the scale of the Nobeyama radio heliograph beam (shown below on the left). The upper V-map present brightness  $T_b^V$ , the lower one - percentage  $P\%$  of polarization.

# Plasma radiation

Plasma oscillations (**Langmuir waves**) are a **natural mode** of a plasma and can be excited by a variety of mechanisms.

In the Sun's corona, the propagation of **electron beams** and/or **shocks** can excite plasma waves.

These are converted from **longitudinal** oscillations to **transverse** oscillation through nonlinear wave-wave interactions.

The resulting transverse waves have frequencies near the **fundamental** or **harmonic** of the local electron plasma frequency: i.e.,  $\nu_{pe}$  or  $2\nu_{pe}$ .

# Plasma radiation

Plasma radiation is therefore thought to involve several steps:

## Fundamental plasma radiation

- A process must occur that is unstable to the production of Langmuir waves
- These must then scatter off of thermal ions or, more likely, low-frequency waves (e.g., ion-acoustic waves)

$$\omega_L + \omega_S = \omega_T$$

and

$$k_L + k_S = k_T$$

coalescence

or

$$\omega_L = \omega_S + \omega_T$$

$$k_L = k_S + k_T$$

decay

# Plasma radiation

Plasma radiation is therefore thought to involve several steps:

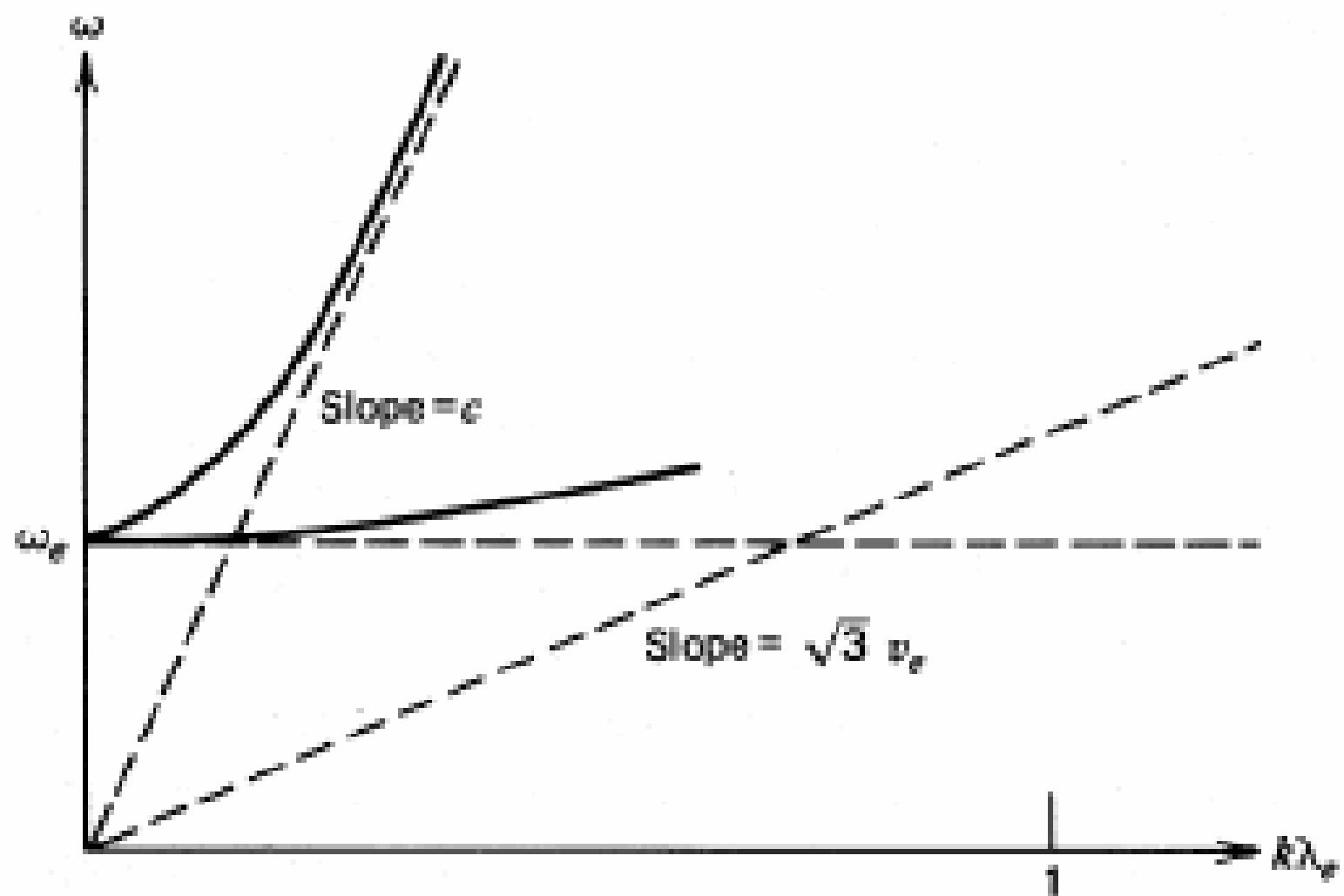
## Harmonic plasma radiation

- A process must occur that is unstable to the production of Langmuir waves
- A **secondary spectrum** of Langmuir waves must be generated
- Two Langmuir waves can then coalesce

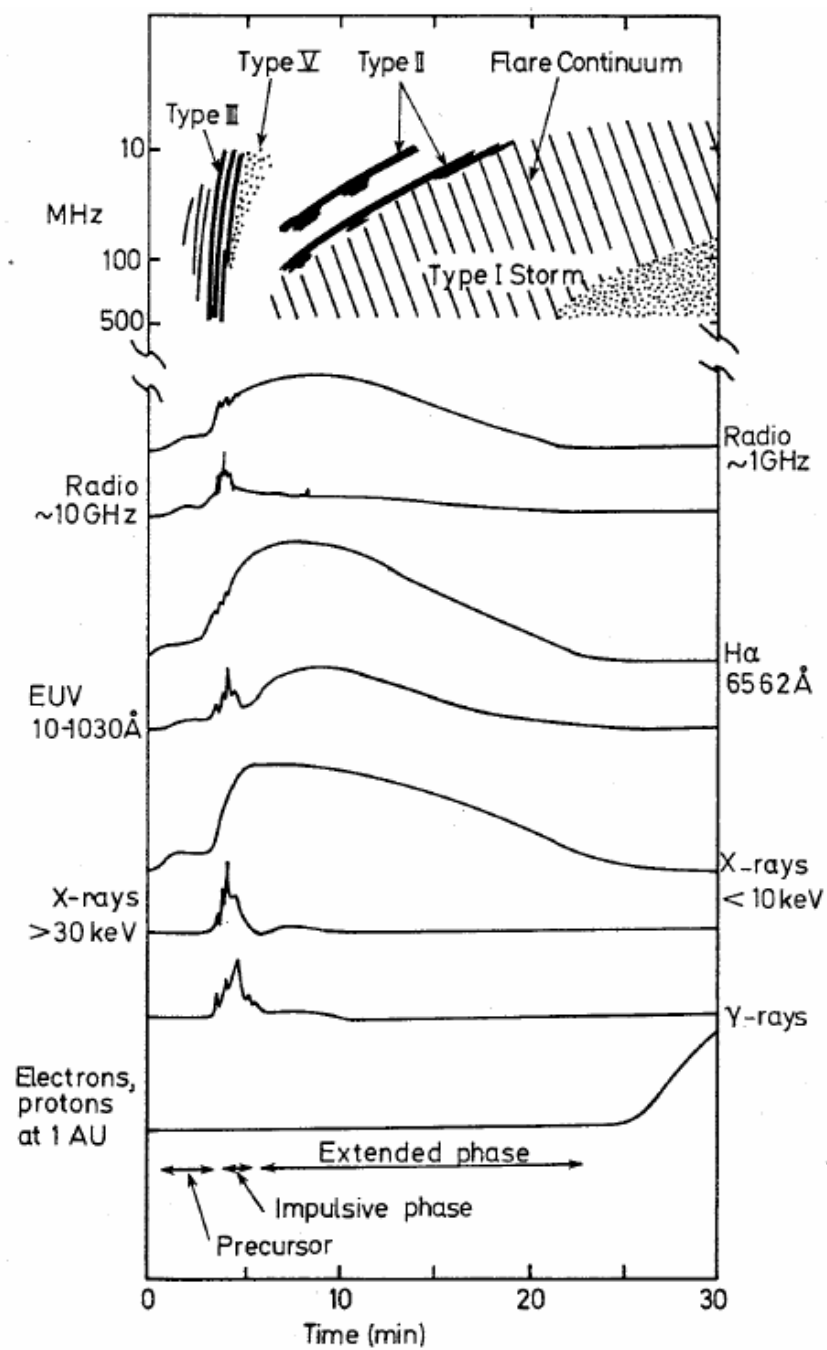
$$\omega_L^1 + \omega_L^2 = \omega_T \quad \text{and} \quad k_L^1 + k_L^2 = k_T \ll k_L$$

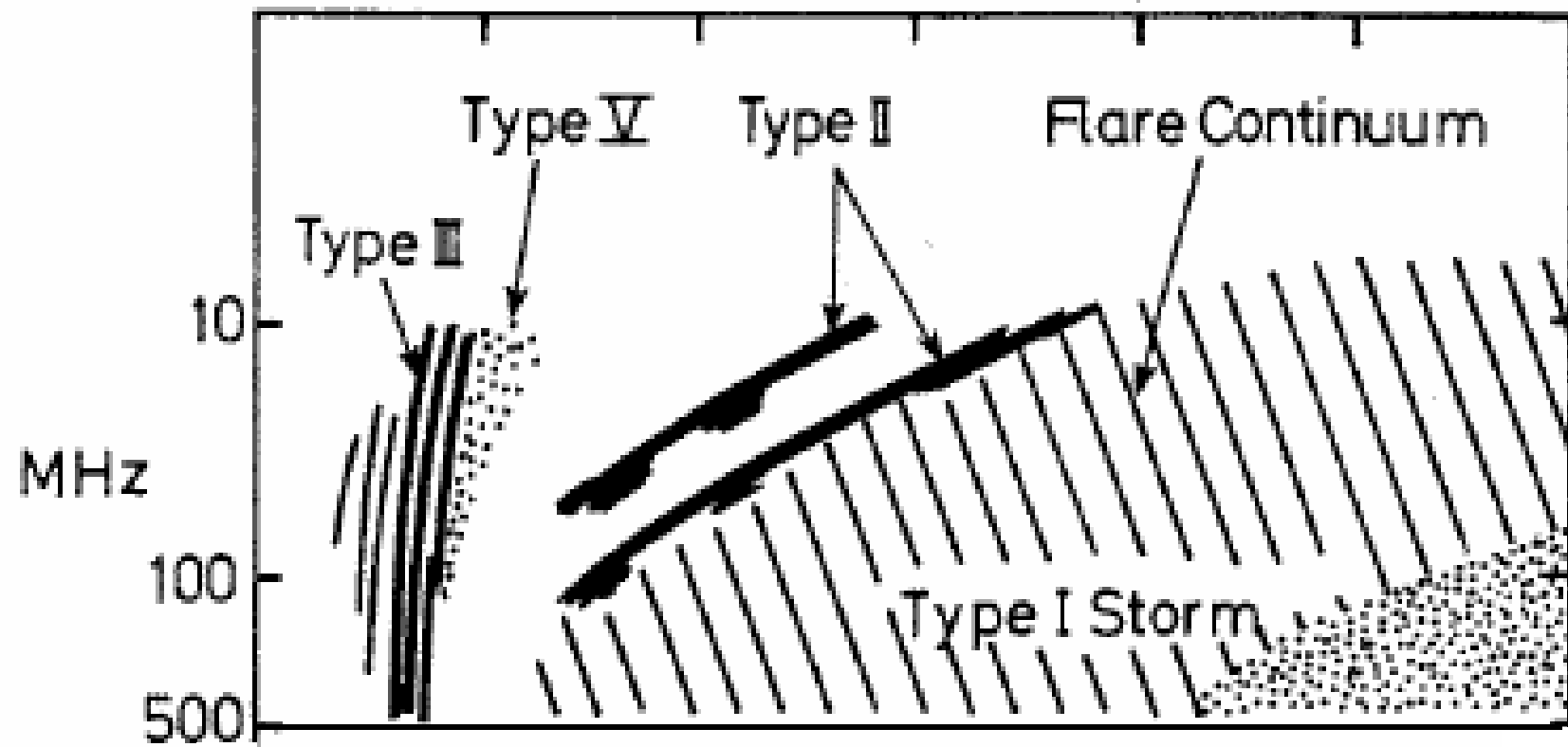
$$\omega_T \approx 2\omega_L$$

$$k_L^1 \approx -k_L^2$$

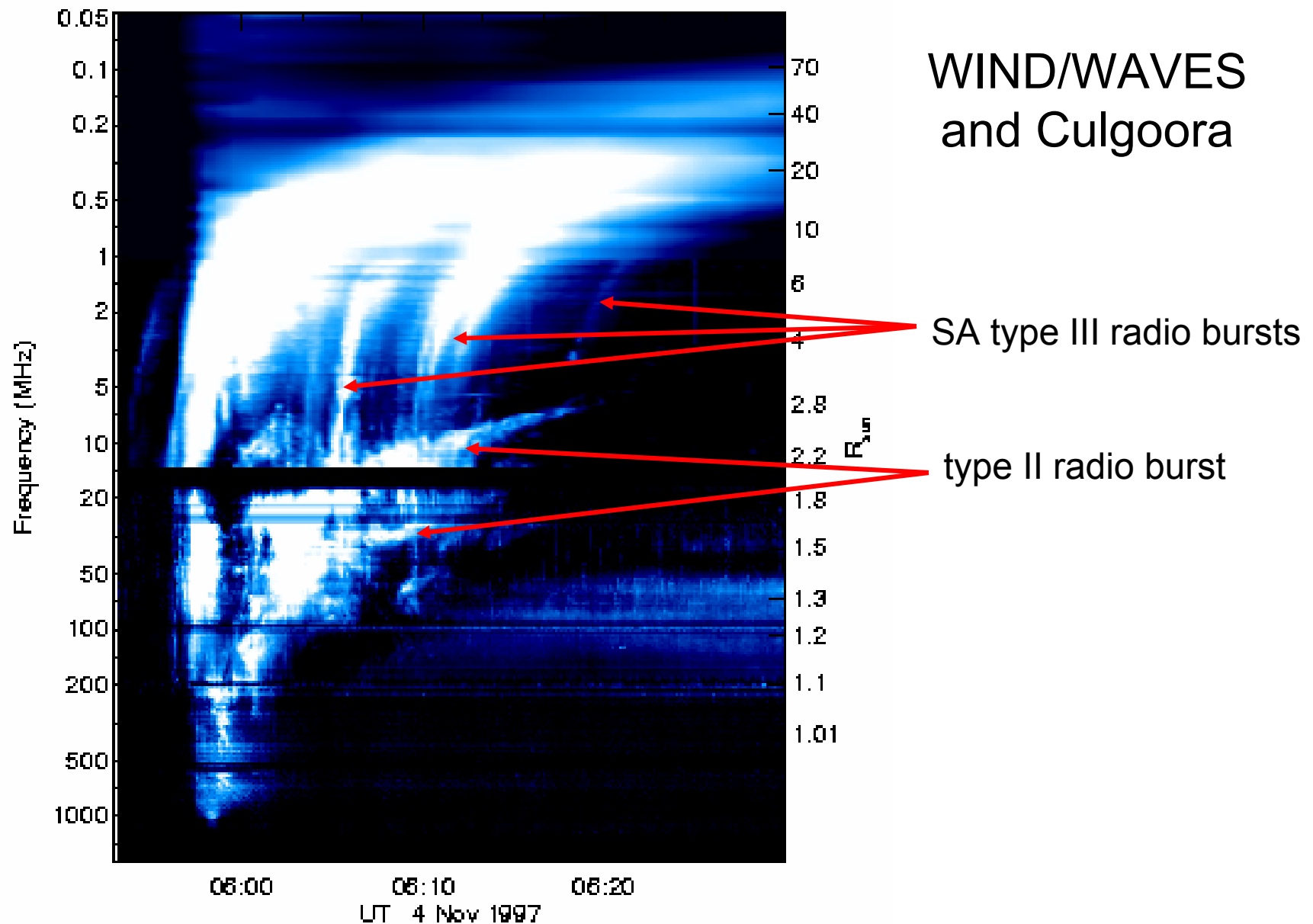




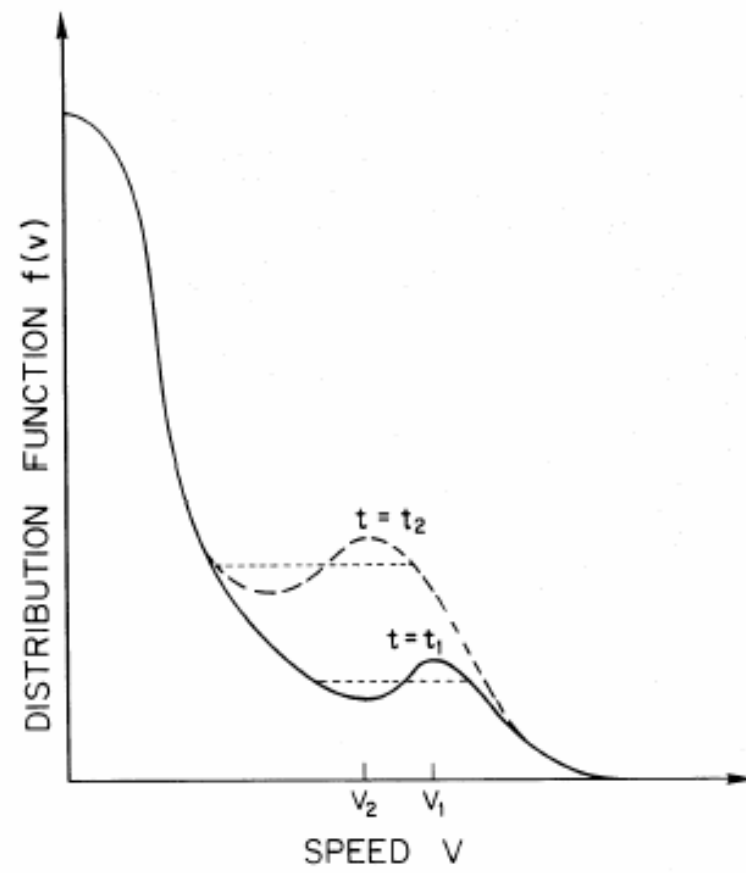




“Classical” radio bursts

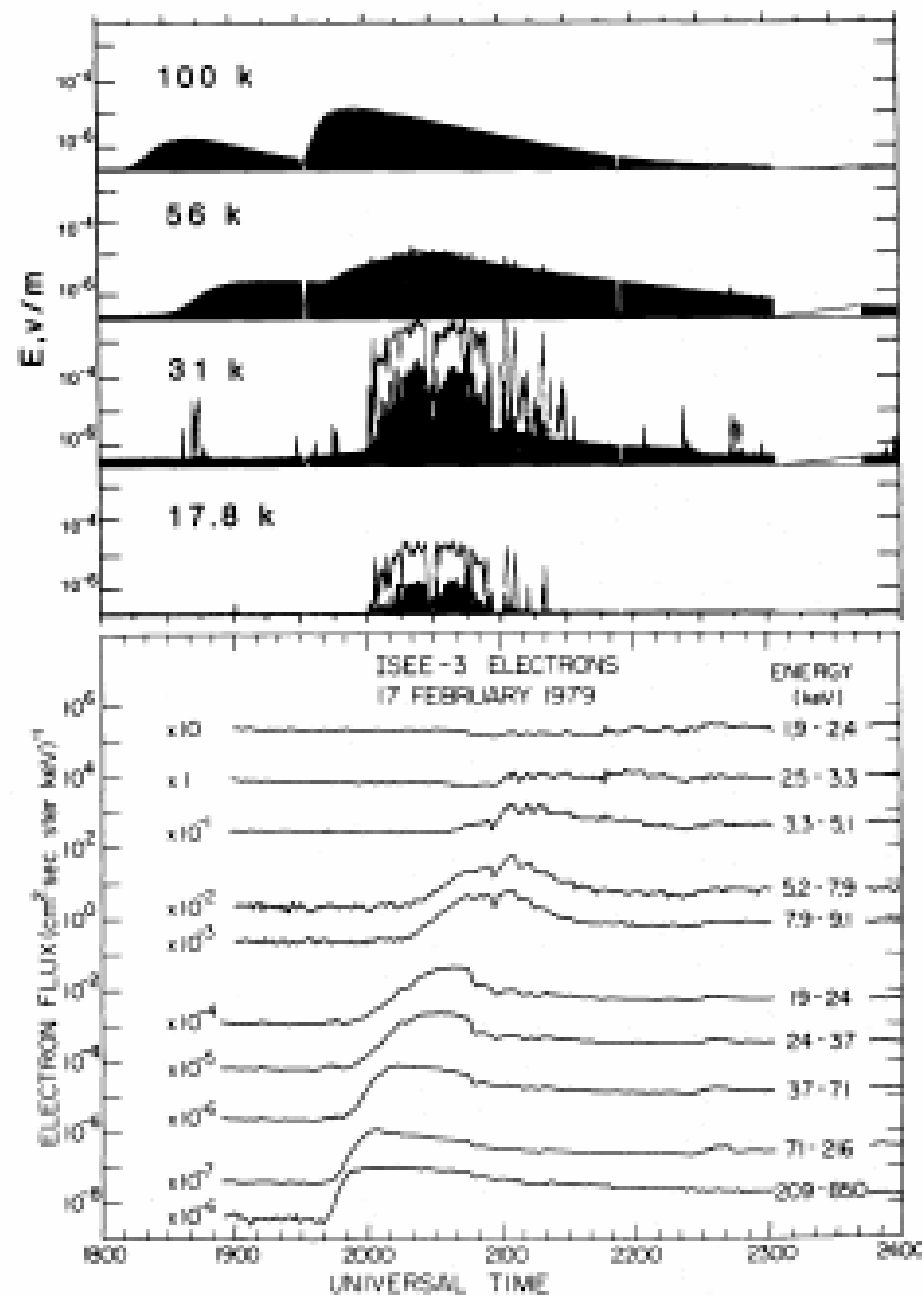


More examples: <http://www.nrao.edu/astrores/gbsrbs>



ISEE-3 type III

1979 Feb 17



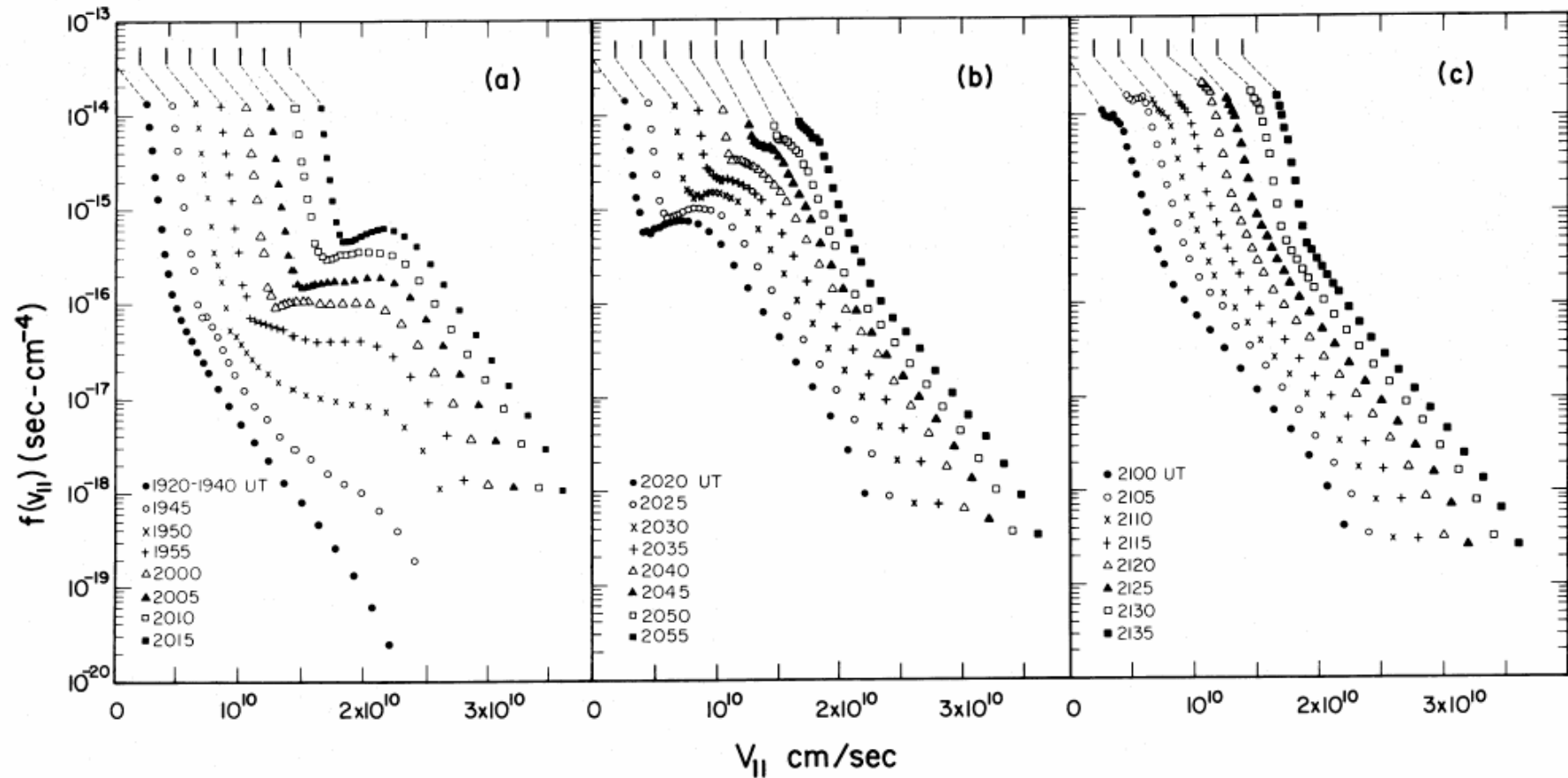
IP Langmuir waves

IP electrons

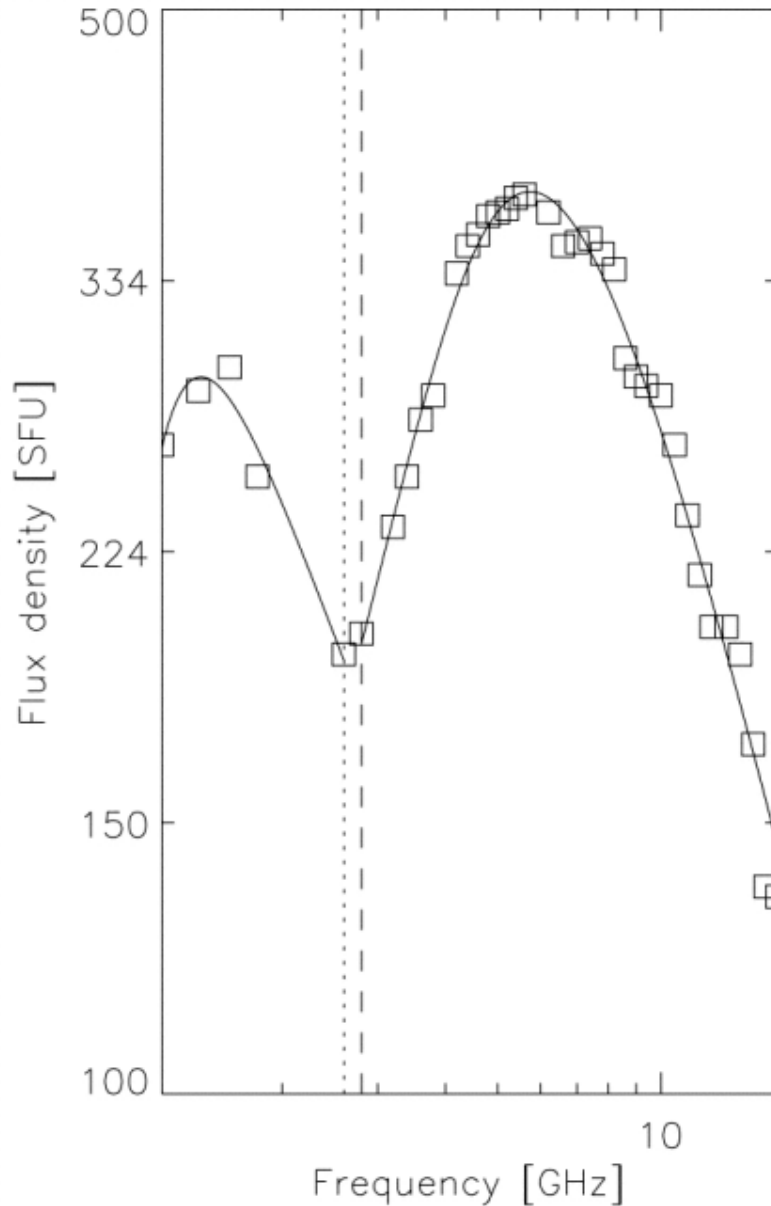
Lin et al. 1981

# ISEE-3 type III

1979 Feb 17



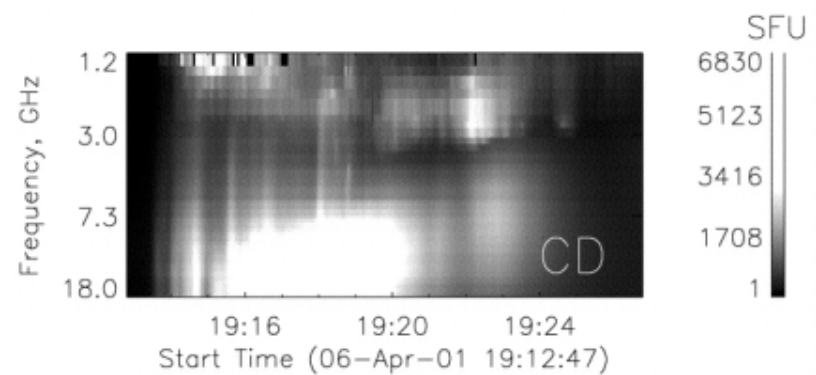
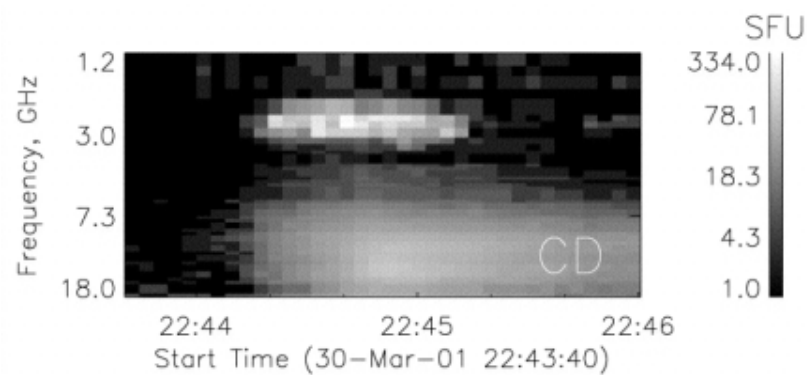
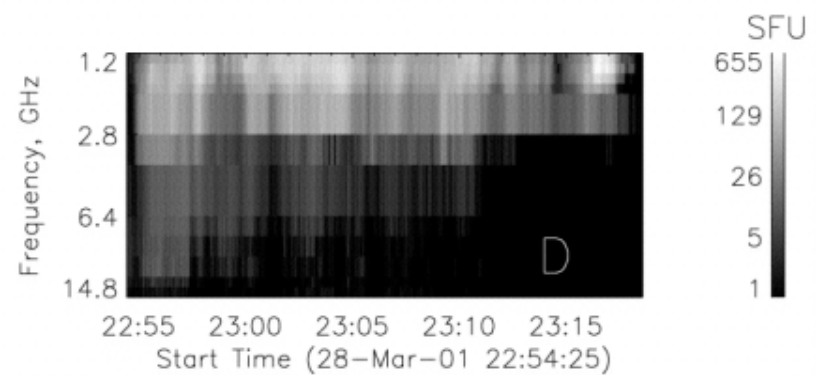
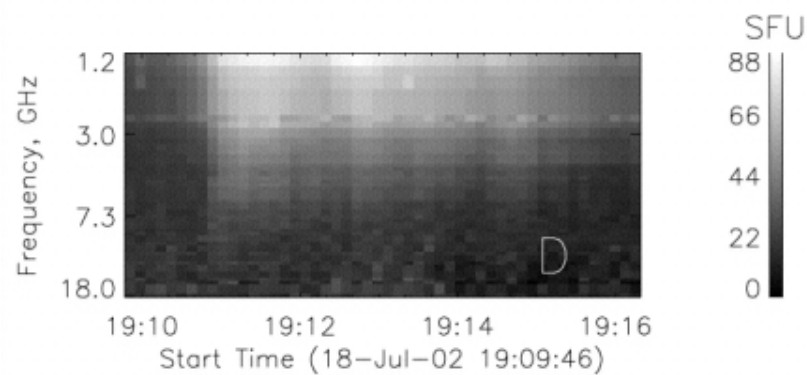
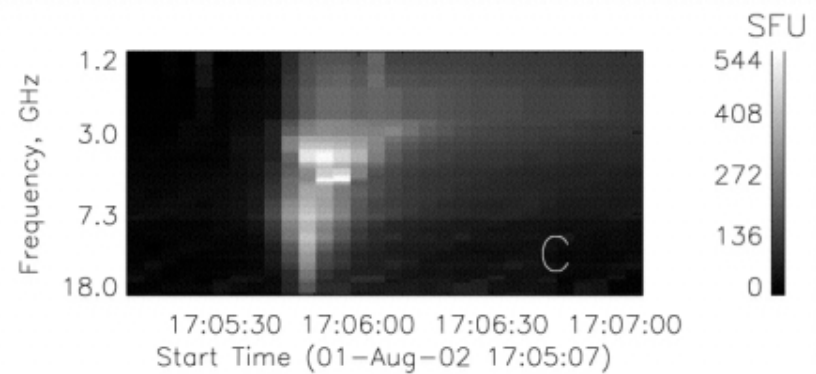
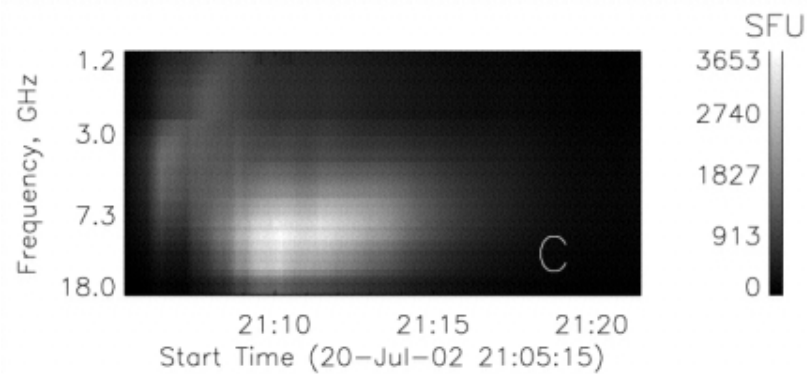
Lin et al. 1981



## Statistical study of spectral properties of dm-cm $\lambda$ radio bursts

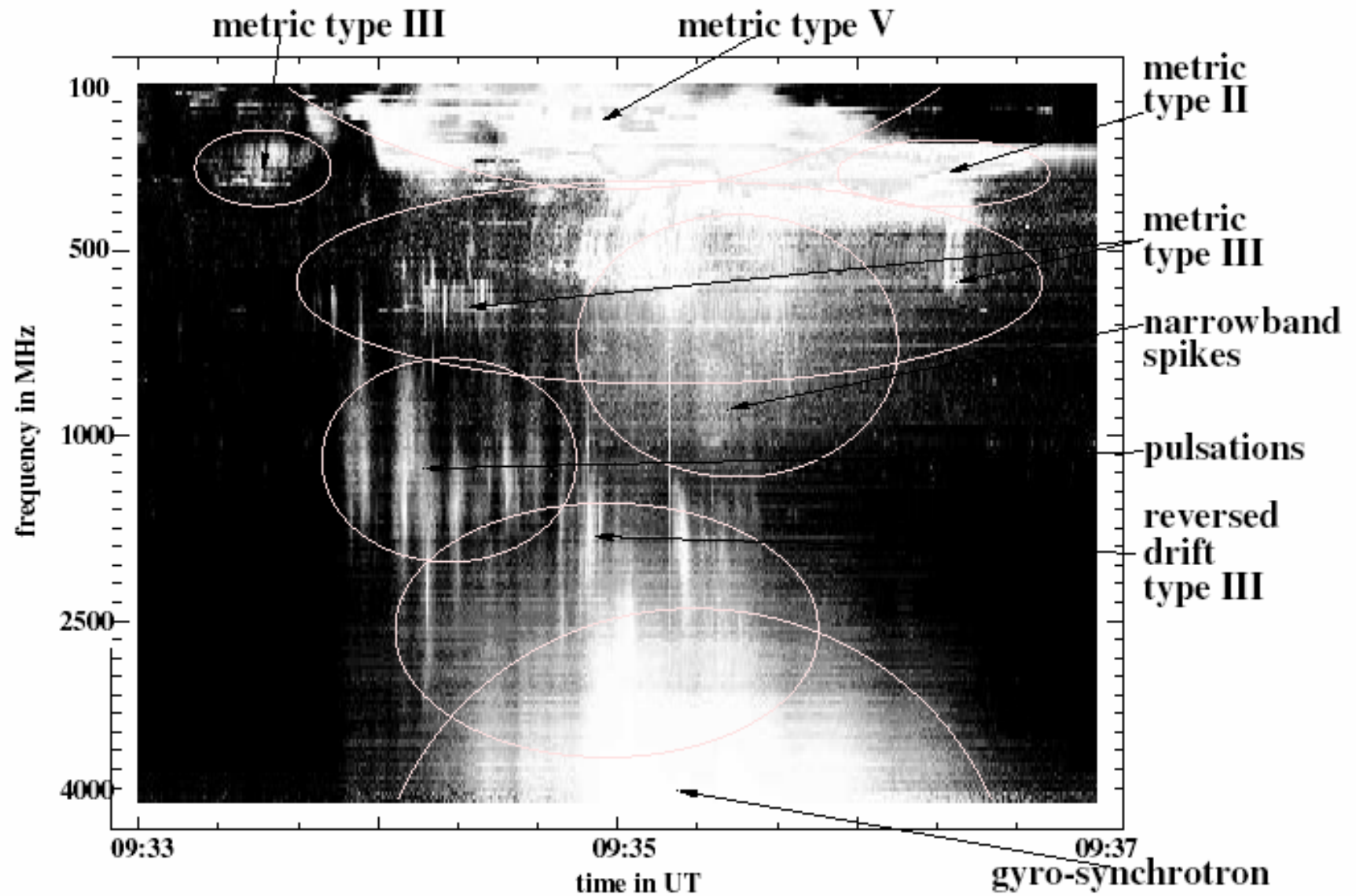
Nita et al 2004

- Sample of 412 OVSA events (1.2-18 GHz)
- Events are the superposition of cm- $\lambda$  ( $>2.6$  GHz) and dm- $\lambda$  ( $<2.6$  GHz) components
- Pure C: 80%; Pure D: 5%; Composite CD: 15%
- For CD events: 12% ( $<100$  sfu); 19% (100-1000 sfu); 60% ( $>1000$  sfu)
- No evidence for harmonic structure

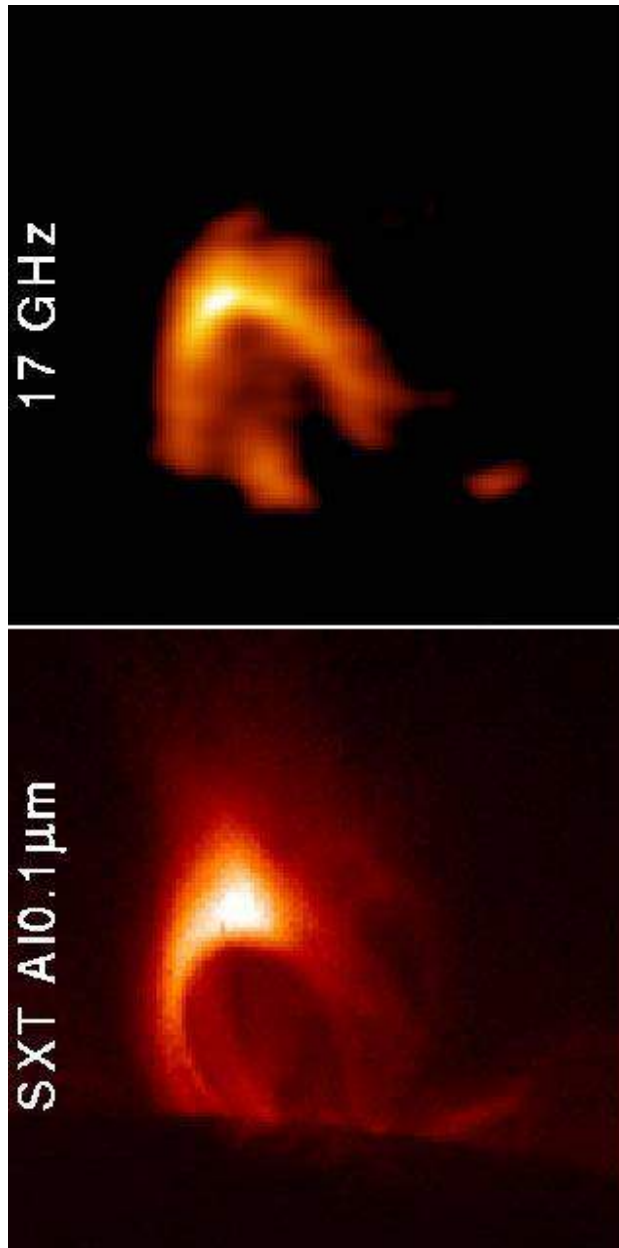


from Nita et al 2004



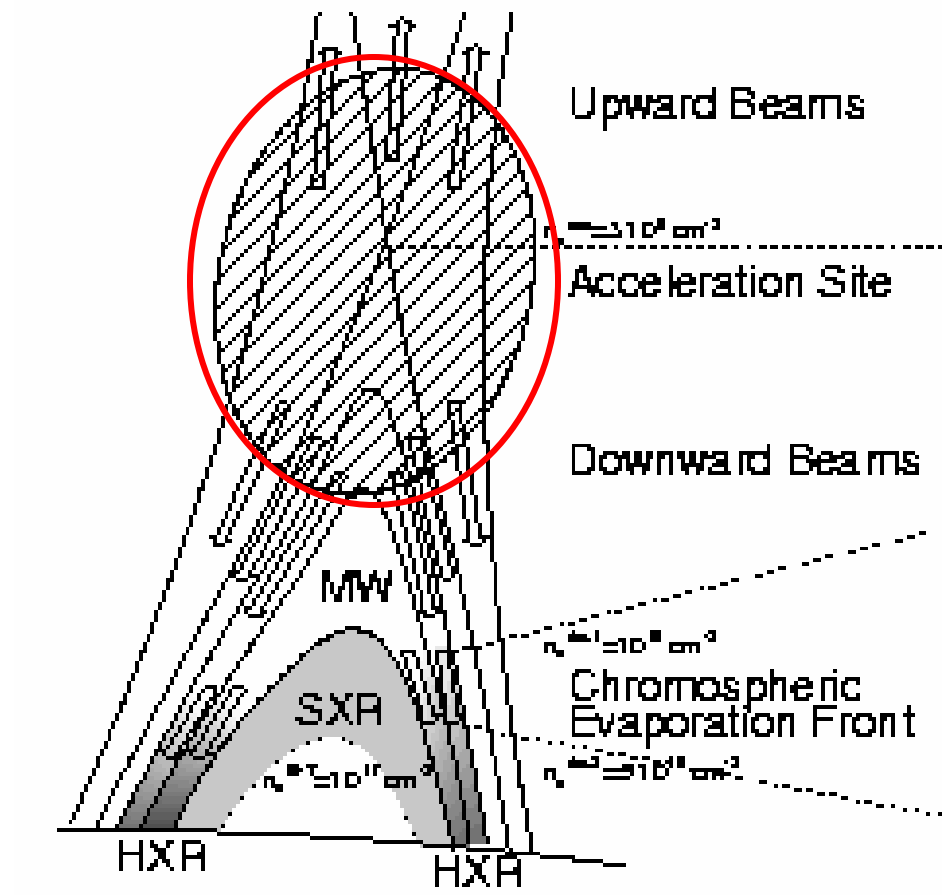


from Benz, 2004



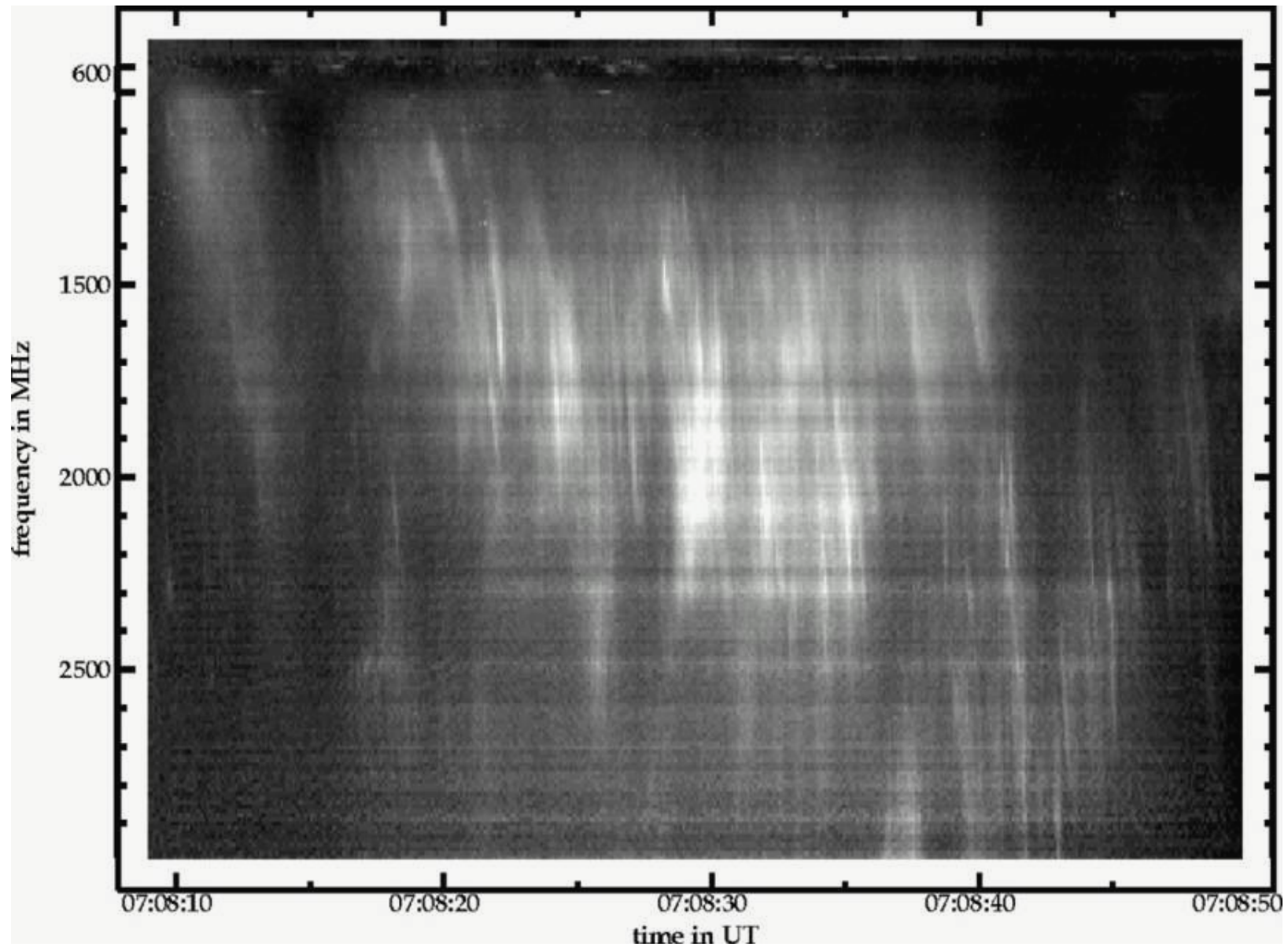
Y. Hanaoka

Long duration flare observed on west limb by Yohkoh and the Nobeyama radioheliograph on 16 March 1993.

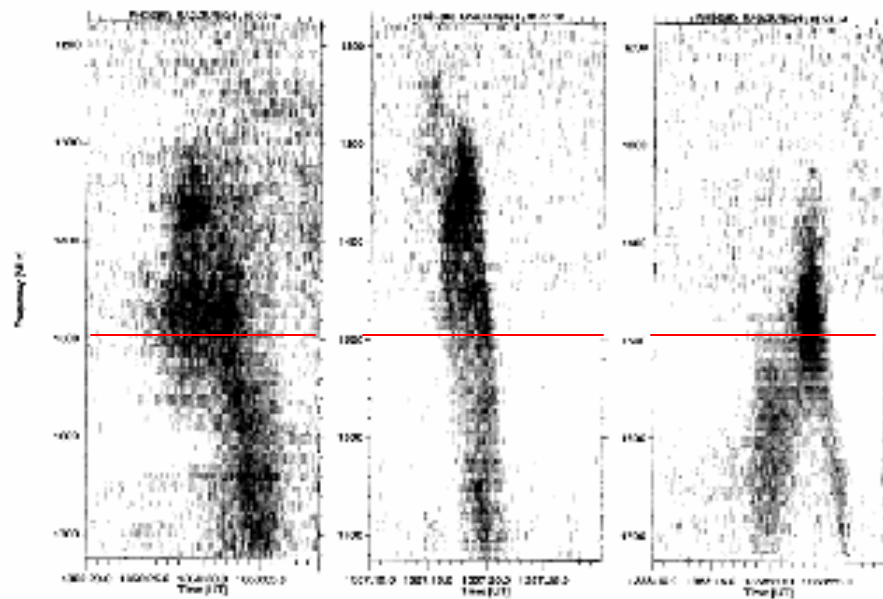


Aschwanden & Benz 1997

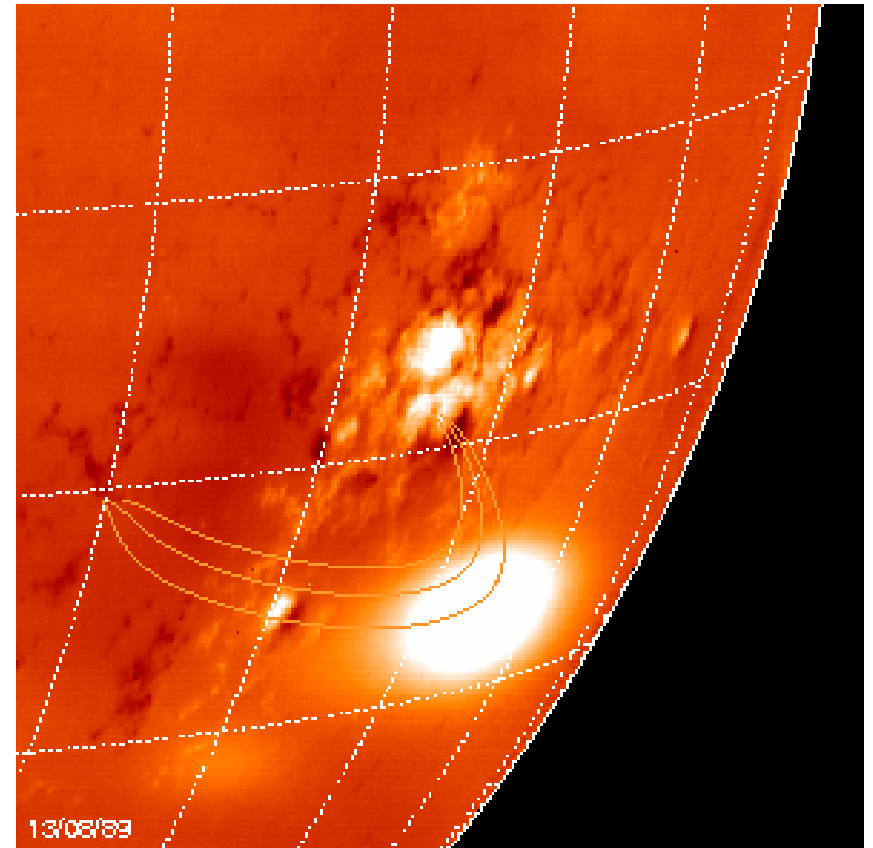
## Reverse slope type III dm radio bursts



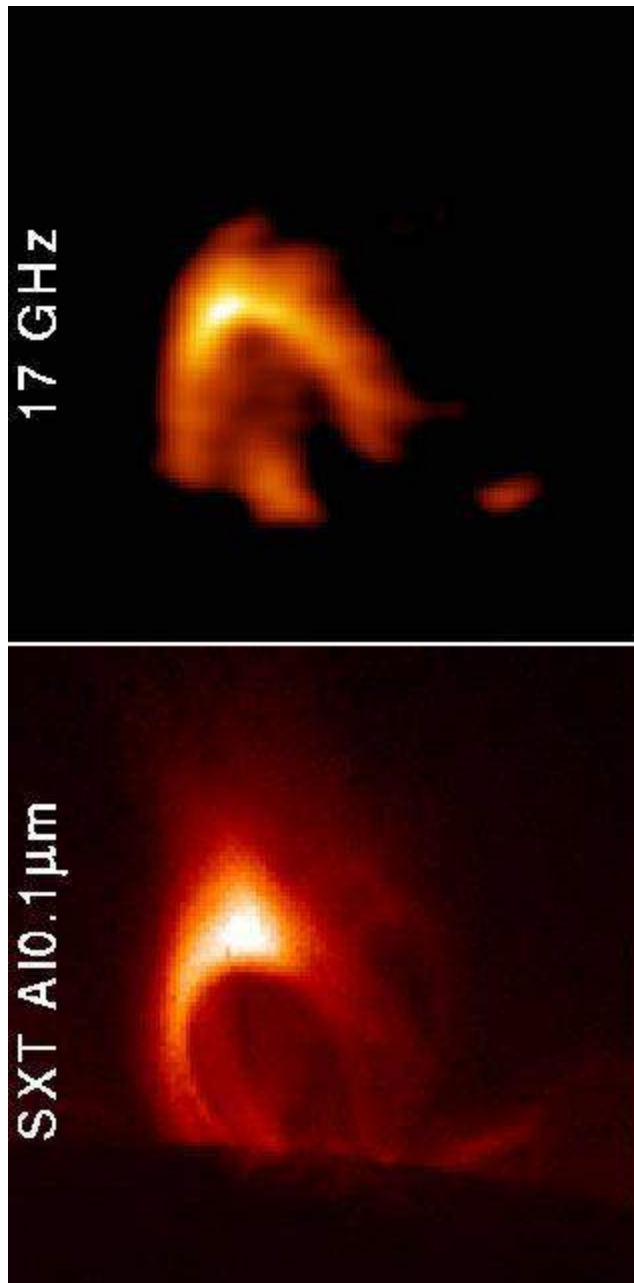
Islaker & Benz 1994



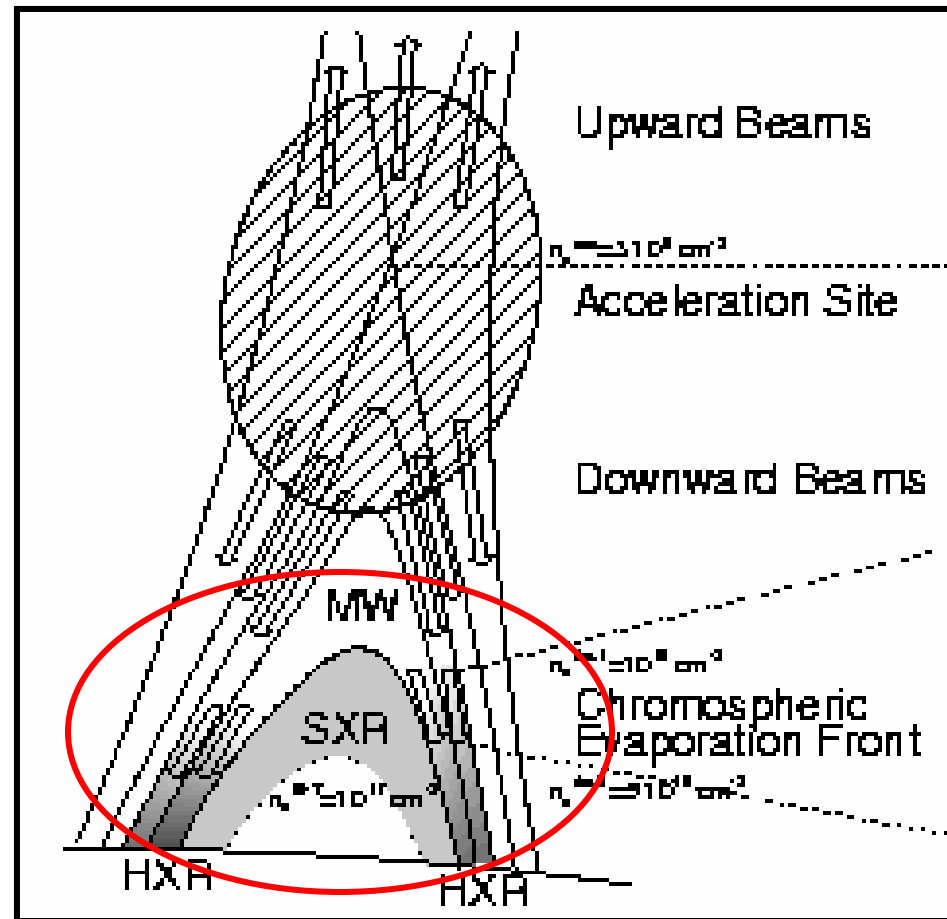
Type U bursts observed by Phoenix/ETH and the VLA.



Aschwanden et al. 1992



Long duration flare observed on west limb by Yohkoh and the Nobeyama radioheliograph on 16 March 1993.



Aschwanden & Benz 1997

# Gyrosynchrotron Radiation

$$I_{\pm}(\nu, \theta) = \frac{j_{\pm}}{\kappa_{\pm}} [1 - \exp(-\kappa_{\pm} D)] \text{ ergs (s sr Hz cm}^2\text{)}^{-1}$$

$$n_{\pm}^2(\theta) = 1 + 2v_p^2(v_p^2 - v^2) \\ \times \{ \pm [v^4 v_b^4 \sin^4 \theta + 4v^2 v_b^2 (v_p^2 - v^2)^2 \cos^2 \theta]^{1/2} \\ - 2v^2(v_p^2 - v^2) - v^2 v_b^2 \sin^2 \theta \}^{-1}.$$

“exact”

Ramaty 1969

Benka & Holman 1992

approximate

Petrosian 1981

Dulk & Marsh 1982, 1985

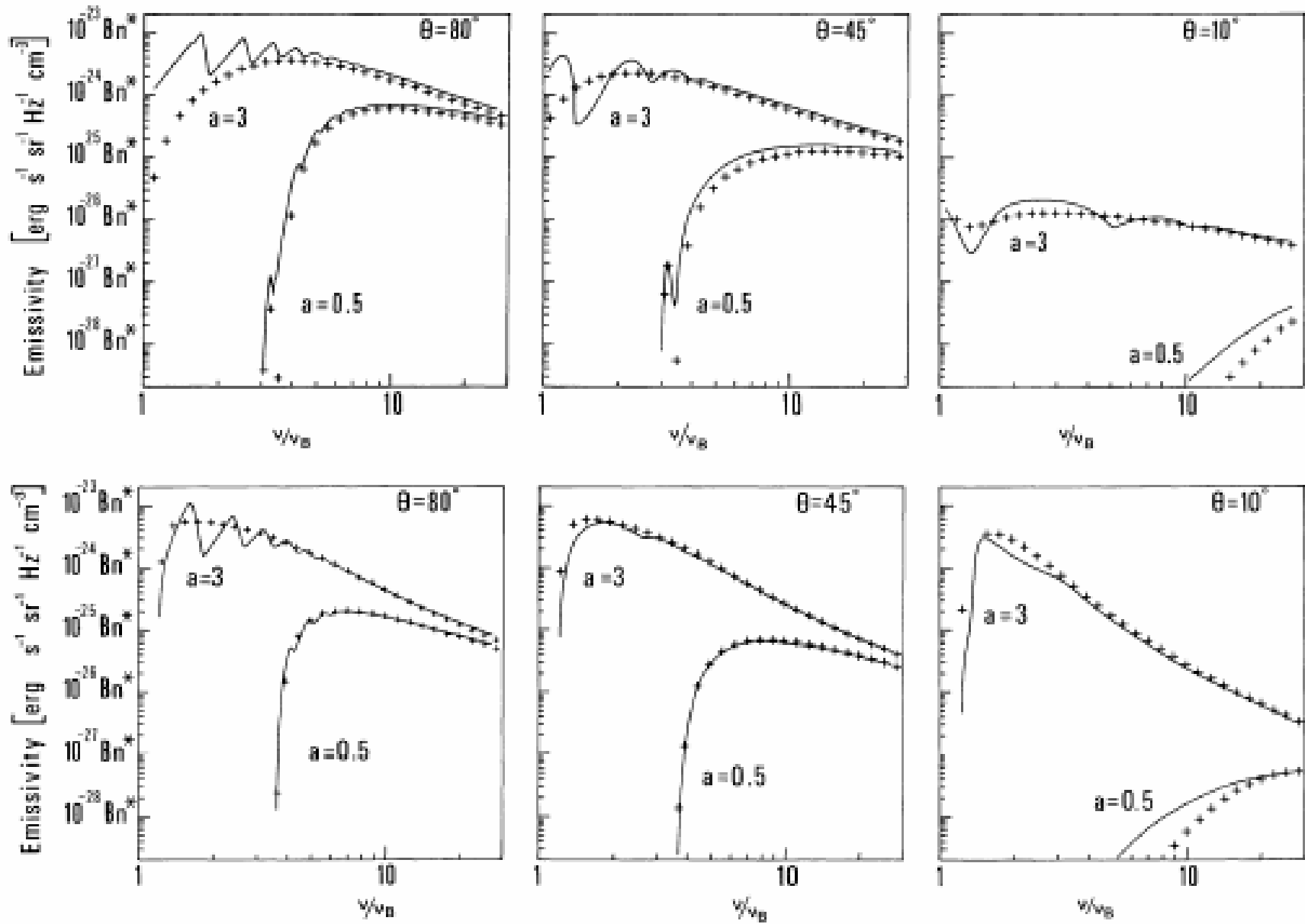
Klein 1987

$$2\pi \int_1^\infty d\gamma \int_{-1}^1 d(\cos \phi) f(\gamma, \phi) = 1$$

$$j_{\pm}(v, \theta) = \frac{4\pi^2 e^2 v N_r}{|\cos \theta| c} \frac{1}{1 + a_{\theta \pm}^2} \sum_{s_{\min}}^{\infty} \int_{\gamma_1}^{\gamma_2} \left\{ d\gamma f(\gamma, \phi_s) \beta^{-1} \right. \\ \left. \times \left[ a_{\theta \pm} \left( \frac{\cot \theta}{n_{\pm}} - \beta \frac{\cos \phi_s}{\sin \theta} \right) J_s(x_s) - \beta \sin \phi_s J'_s(x_s) \right]^2 \right\},$$

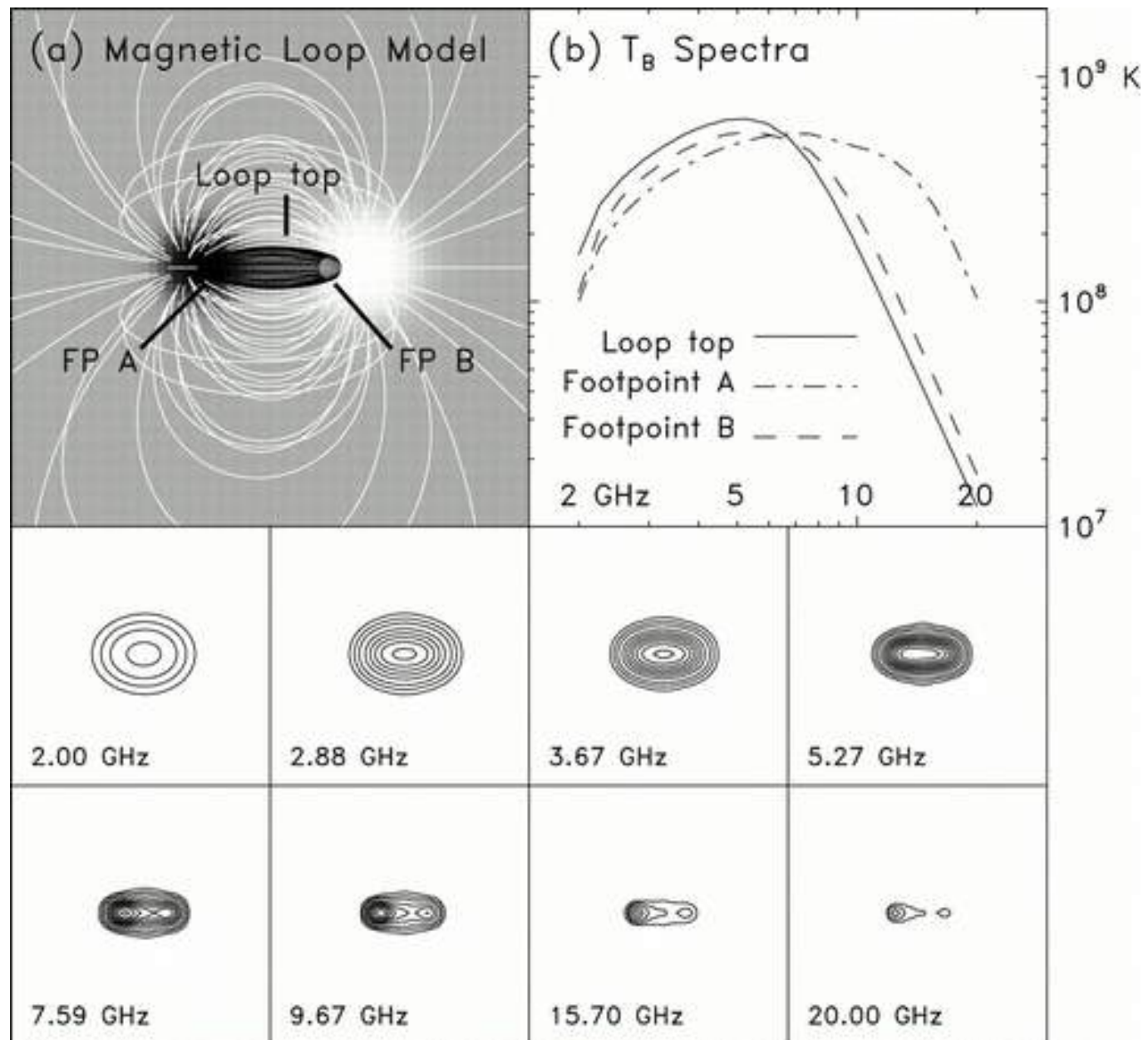
$$\kappa_{\pm}(v, \theta) = \frac{4\pi^2 e^2 N_r}{mcv |\cos \theta|} \frac{1}{n_{\pm} (1 + a_{\theta \pm}^2)} \sum_{s_{\min}}^{\infty} \int_{\gamma_1}^{\gamma_2} d\gamma \beta^{-1} \\ \times \left\{ -\beta \gamma^2 \frac{\partial}{\partial \gamma} \left[ \frac{f(\gamma, \phi)}{\beta \gamma^2} \right] + \frac{n_{\pm} \beta \cos \theta - \cos \phi}{\beta^2 \gamma \sin \phi} \frac{\partial}{\partial \phi} f(\gamma, \phi) \right\}_{\phi=\phi_s} \\ \times \left[ a_{\theta \pm} \left( \frac{\cot \theta}{n_{\pm}} - \beta \frac{\cos \phi_s}{\sin \theta} \right) J_s(x_s) - \beta \sin \phi_s J'_s(x_s) \right]^2$$

e.g., Klein (1987)

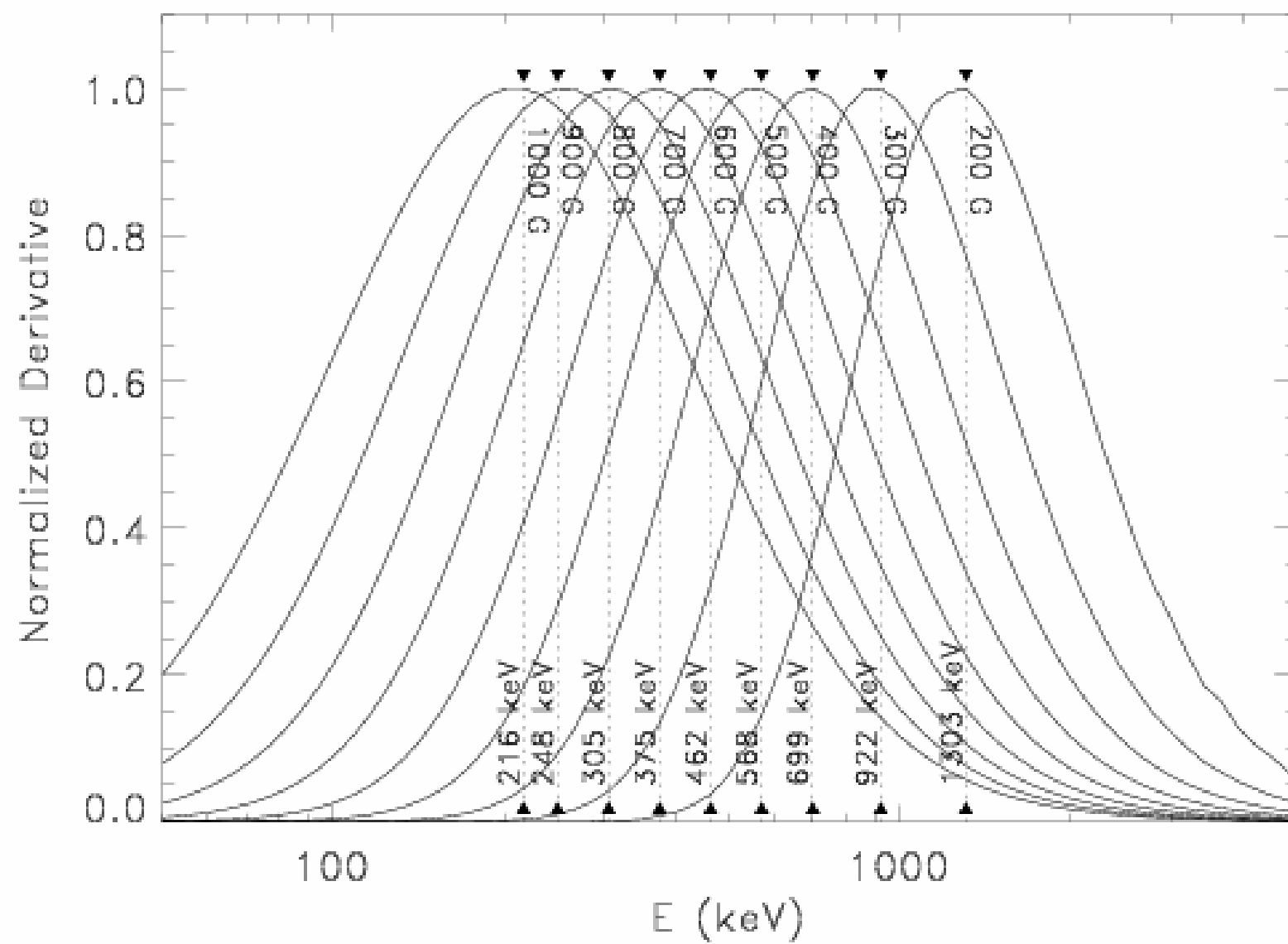


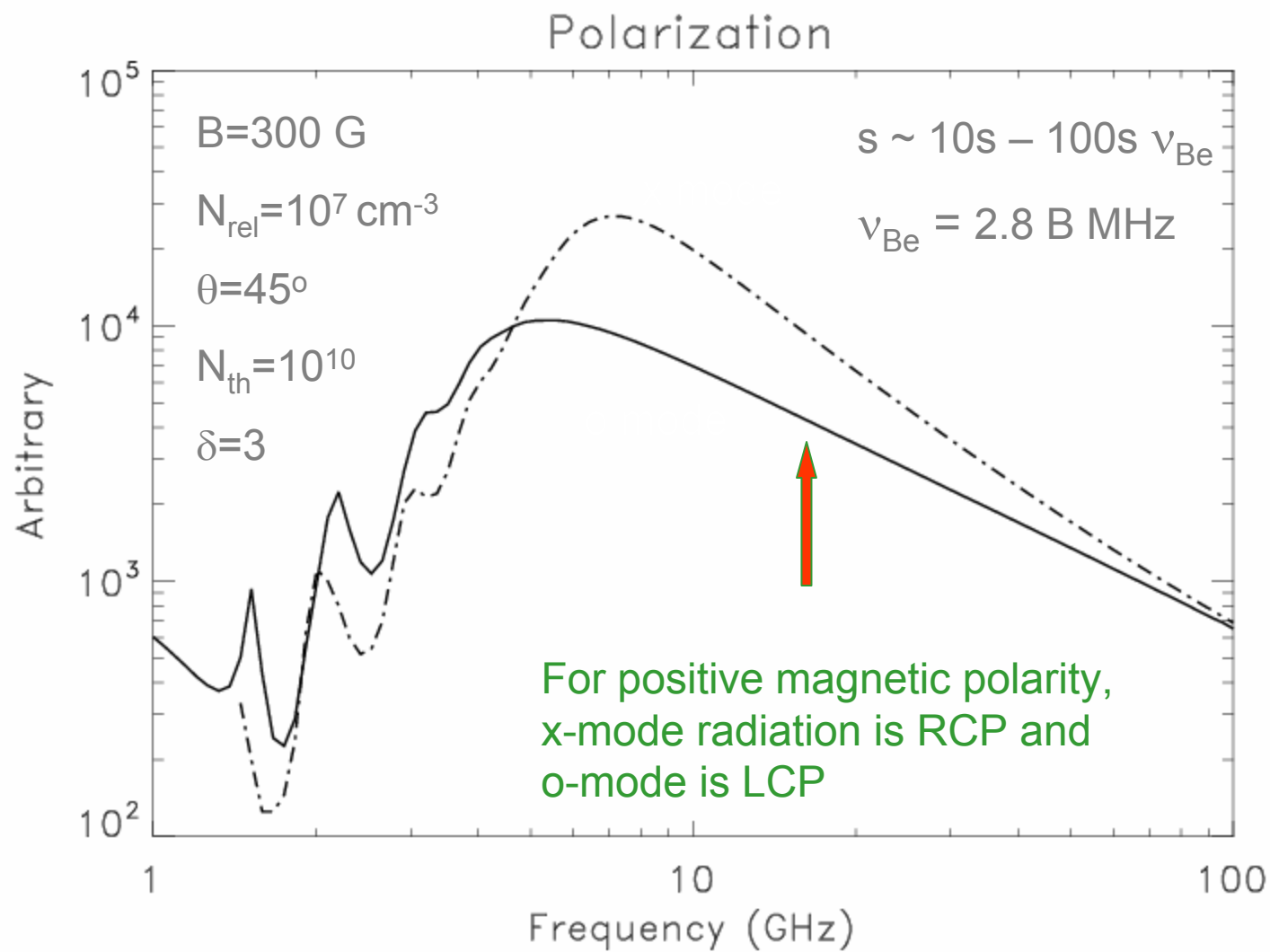


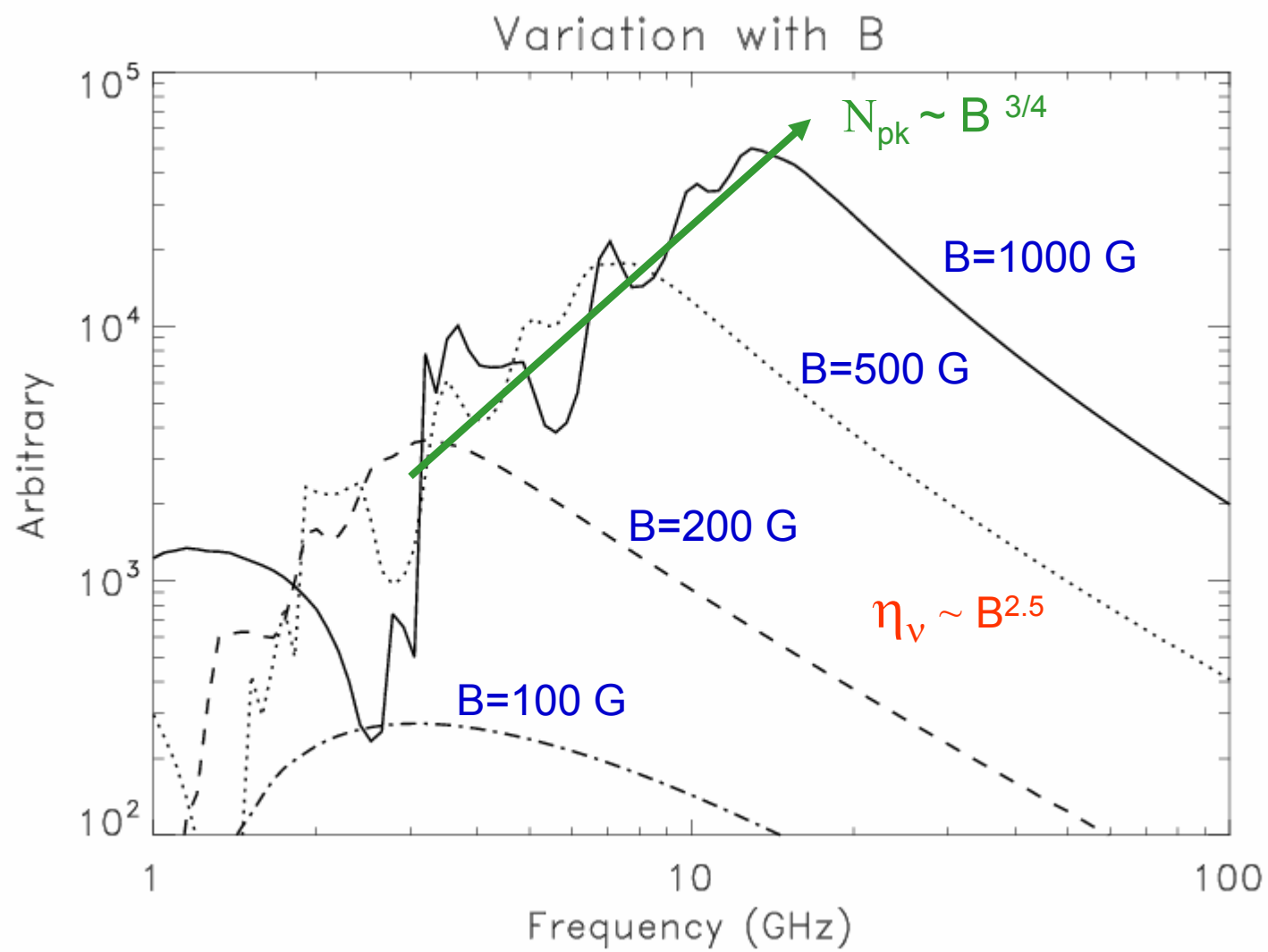
# A Schematic Model

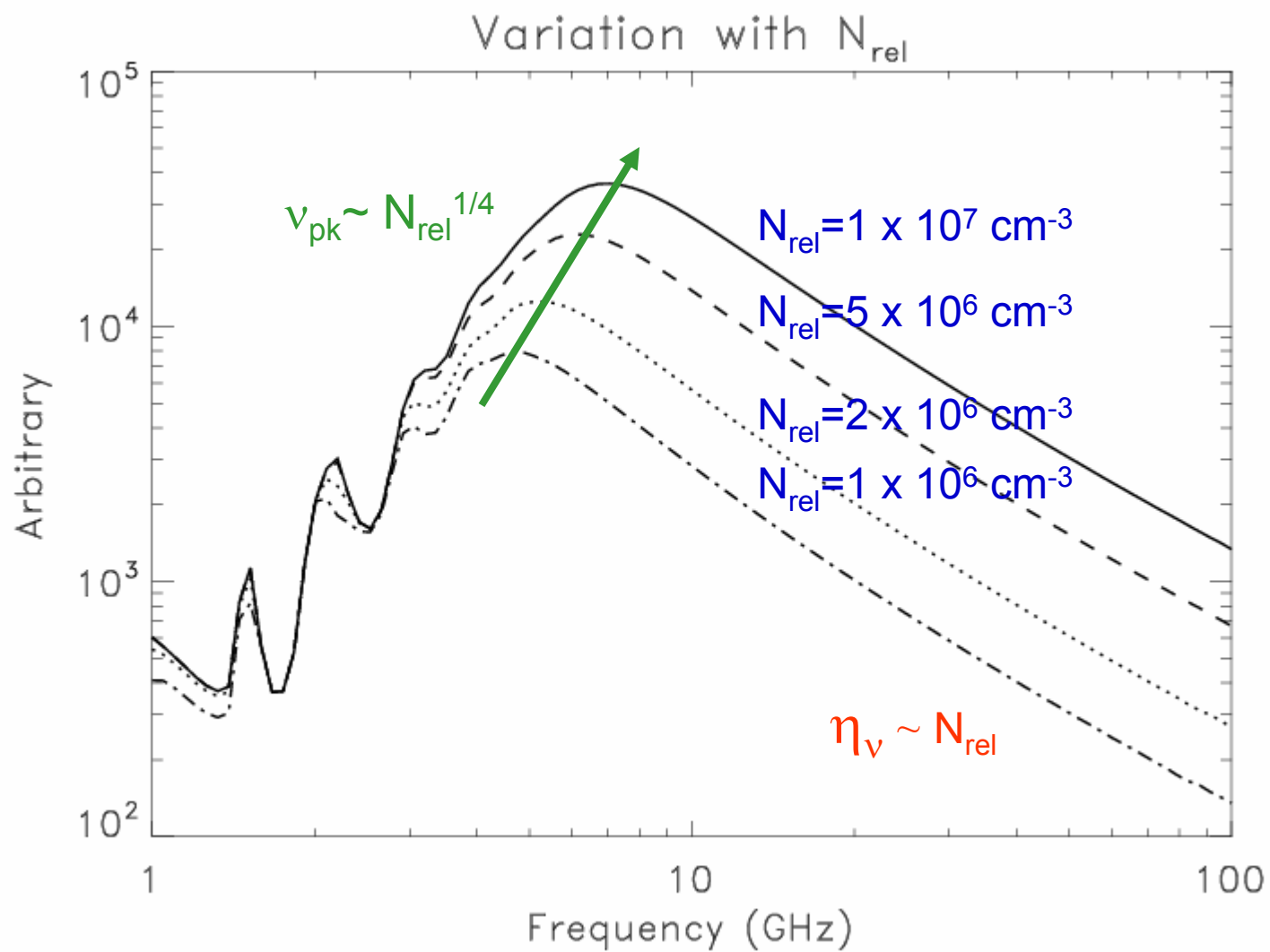


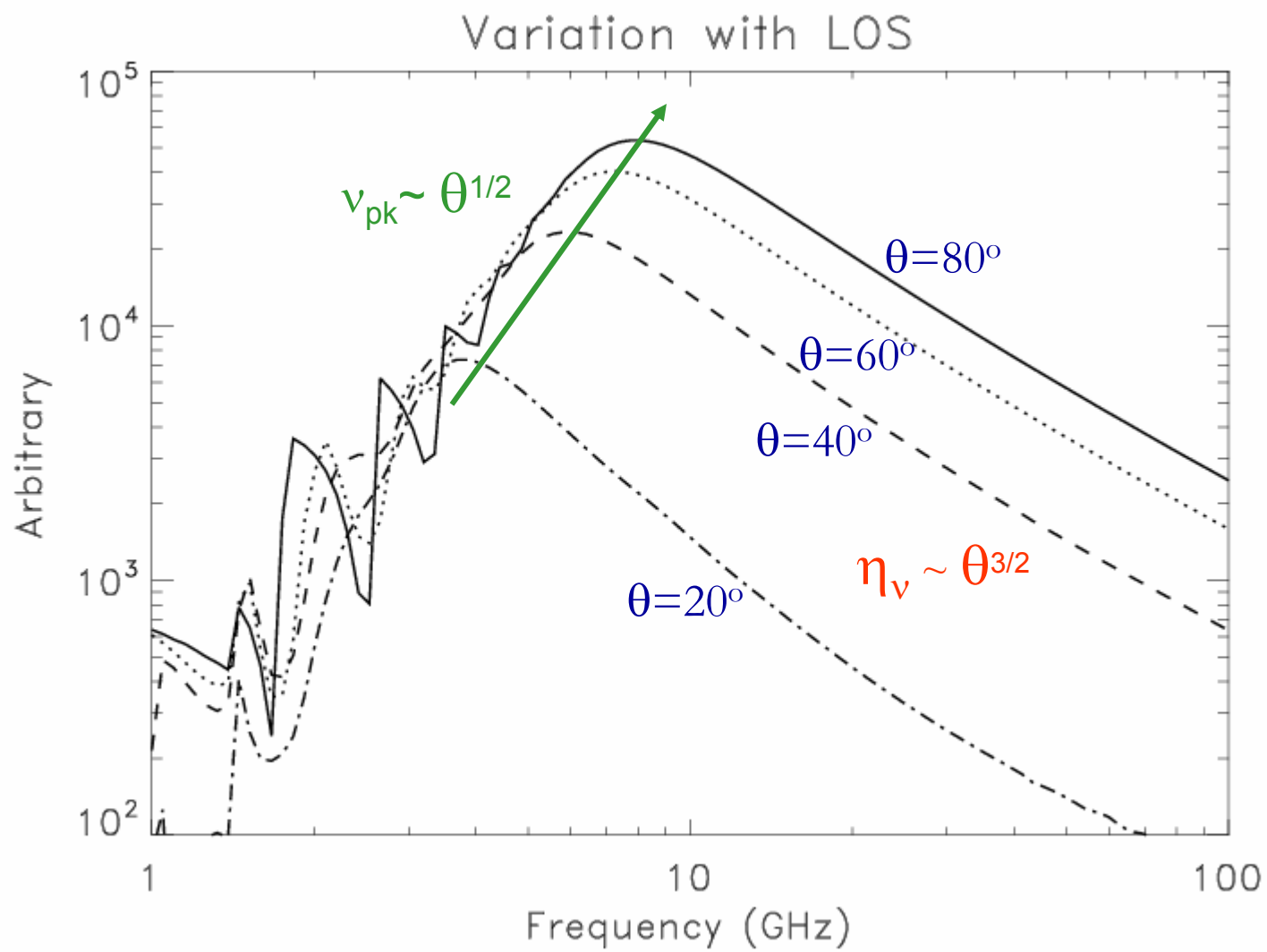
Bastian et al 1998

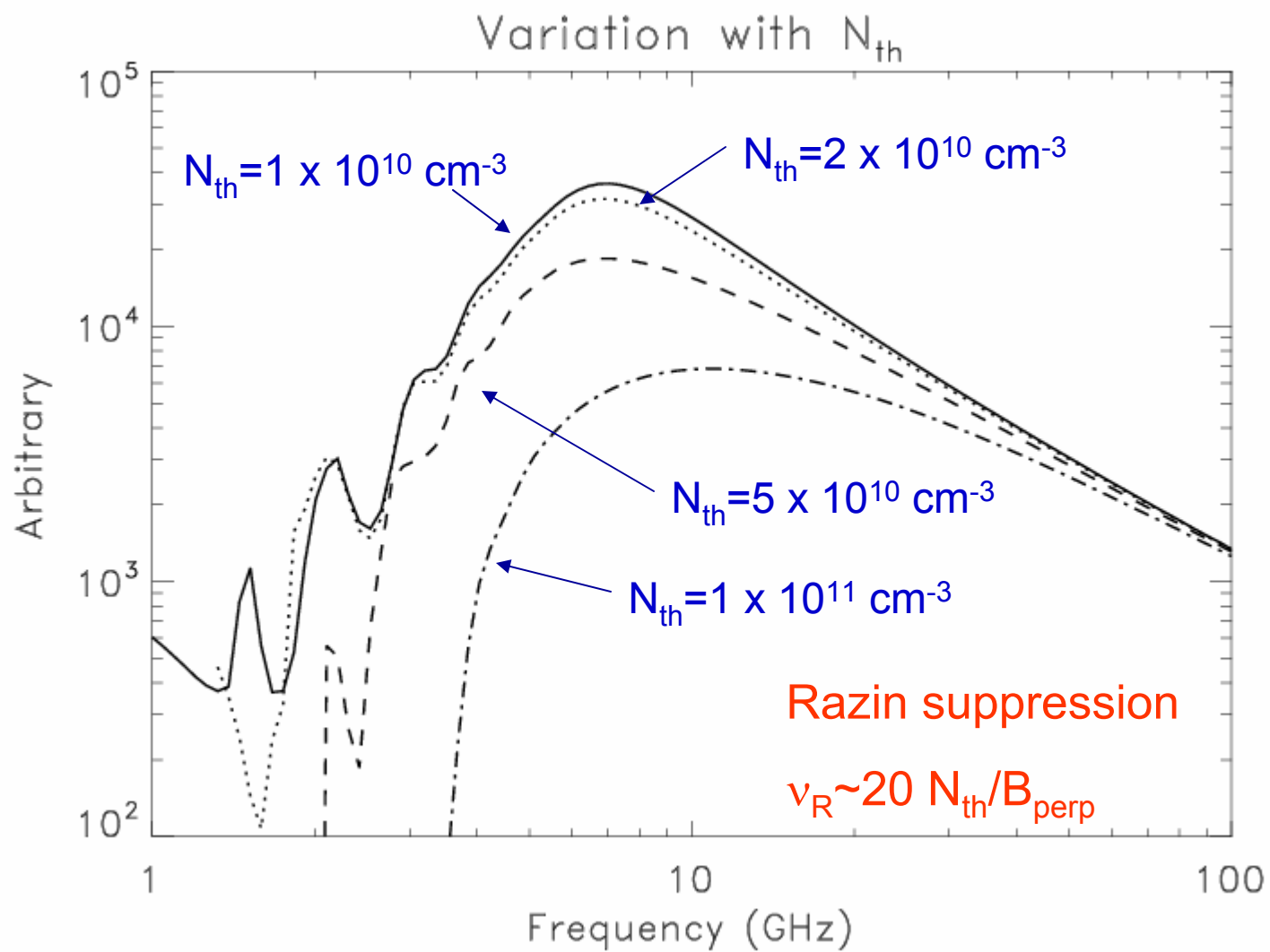












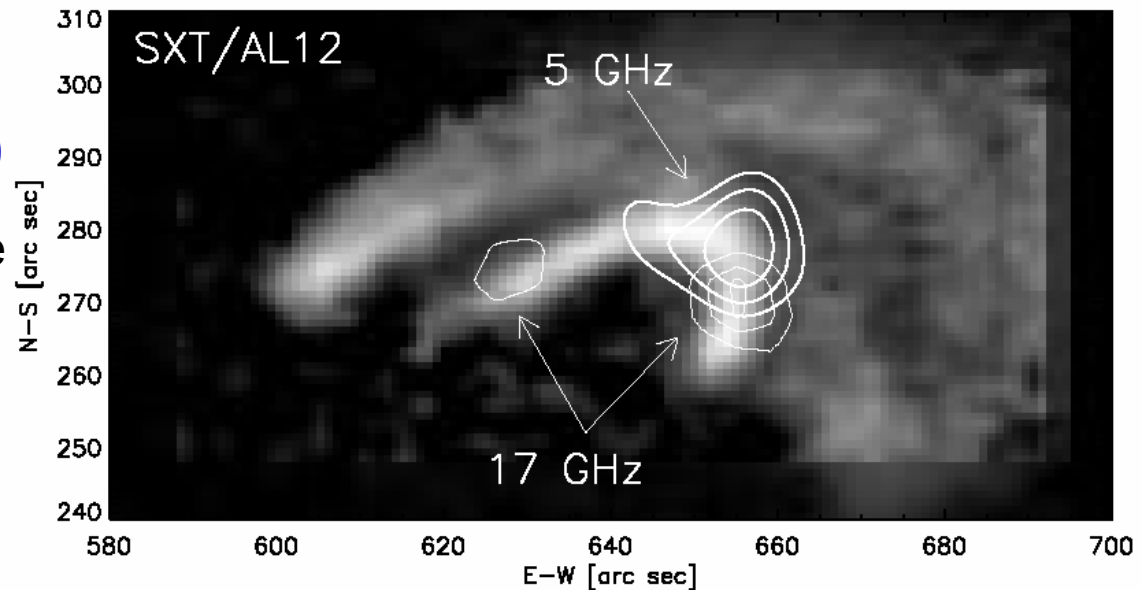
1993 June 3

OVSA/NoRH

## Trap properties

Lee, Gary, & Shibasaki 2000

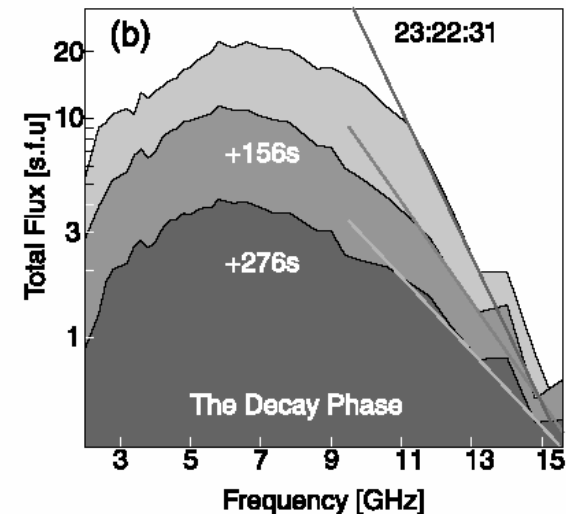
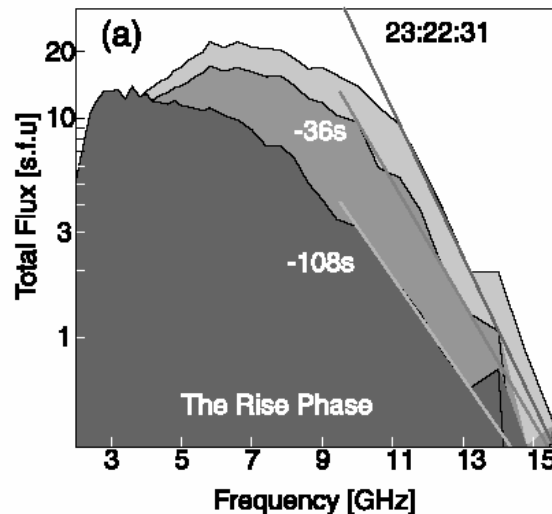
A comparison of successive flares yielded trap densities of  $5 \times 10^9 \text{ cm}^{-3}$  in the first, and  $8 \times 10^{10} \text{ cm}^{-3}$  in the second.



## Anisotropic injection

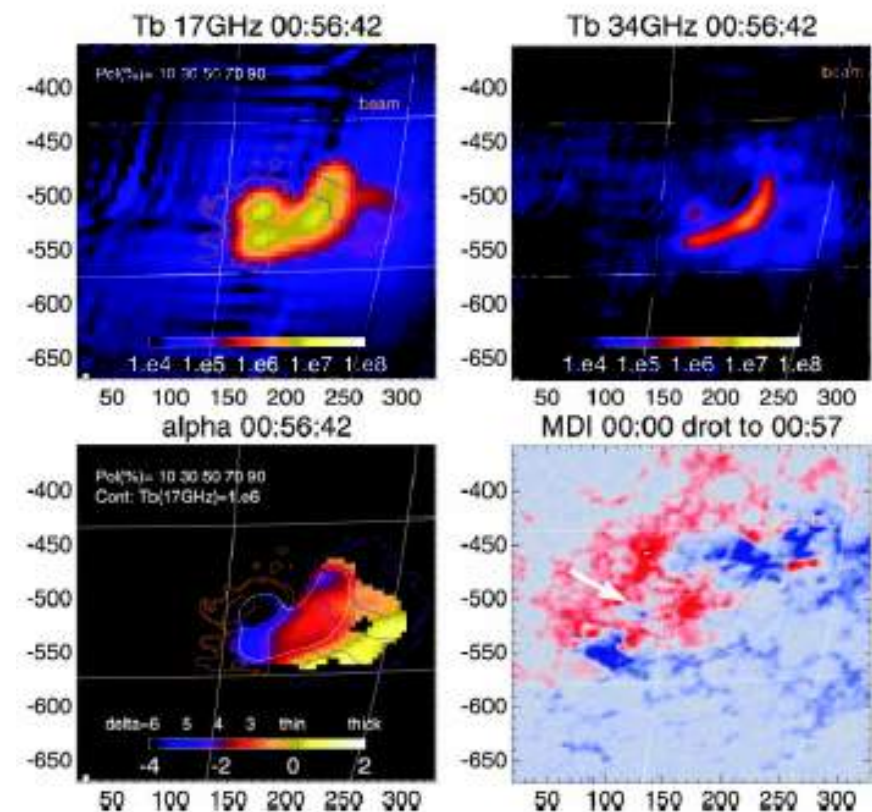
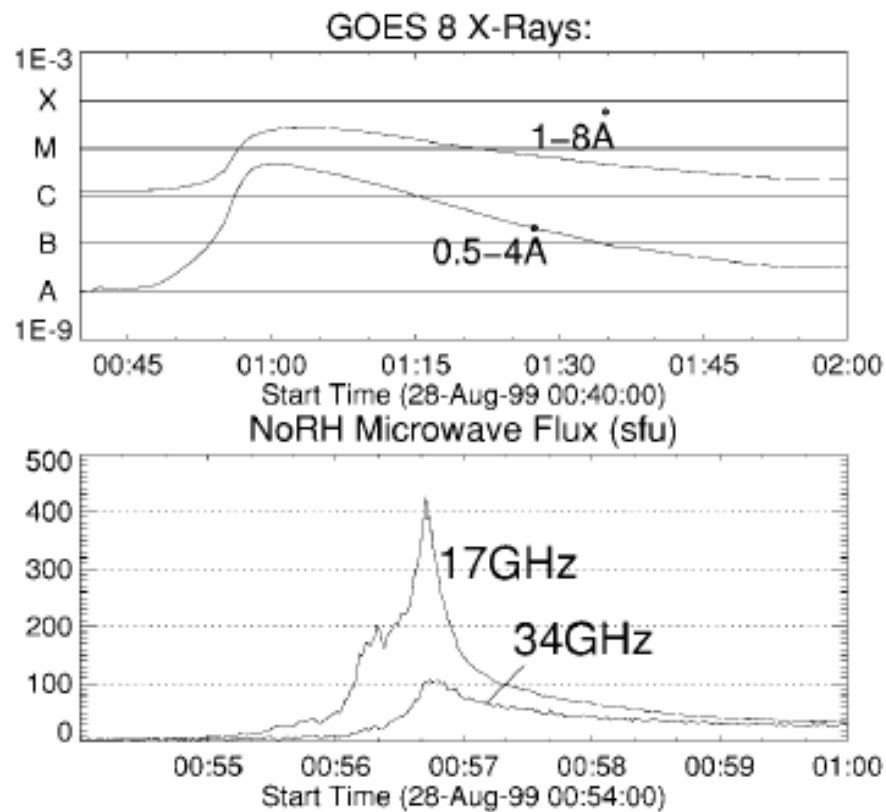
Lee & Gary 2000

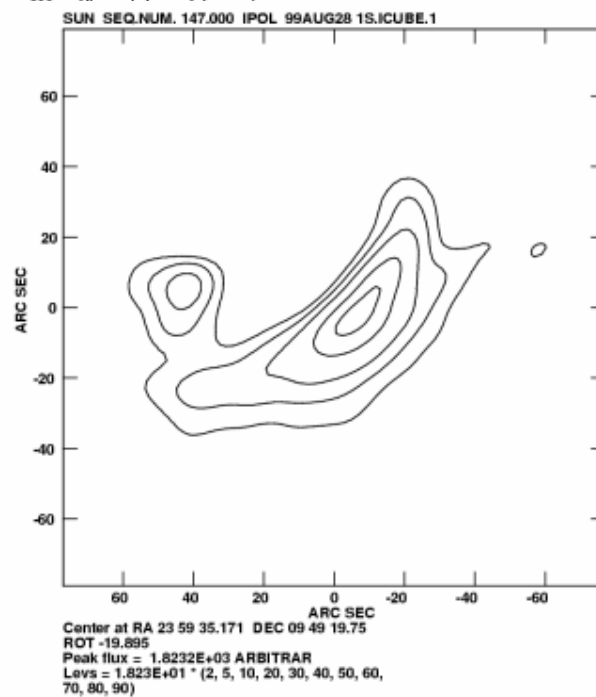
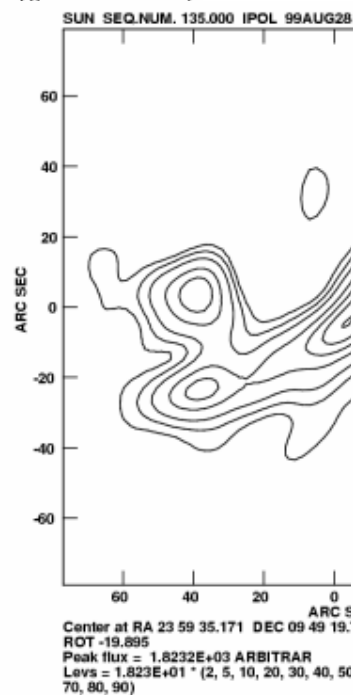
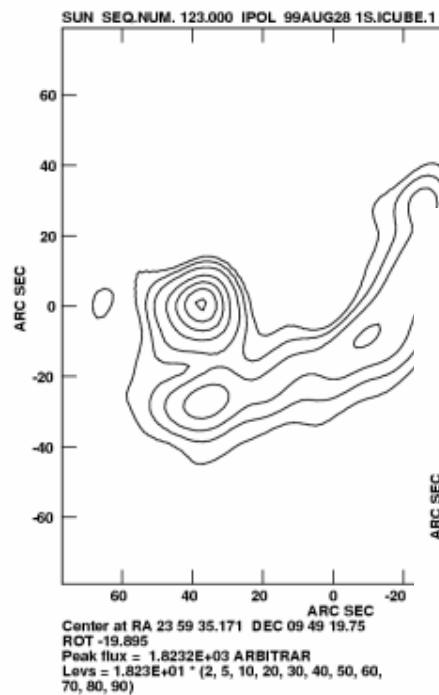
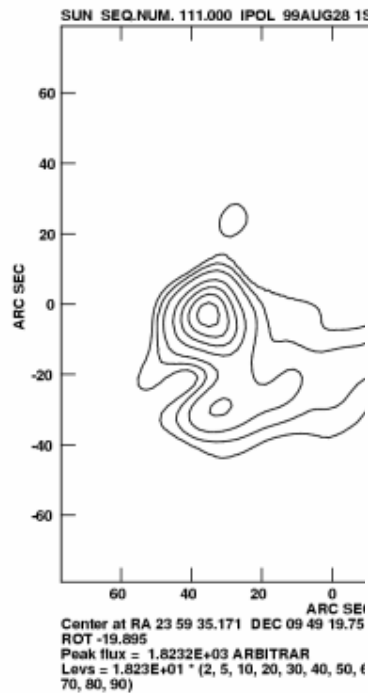
Showed that the electron injection in the first flare was best fit by a **beamed** pitch angle distribution.

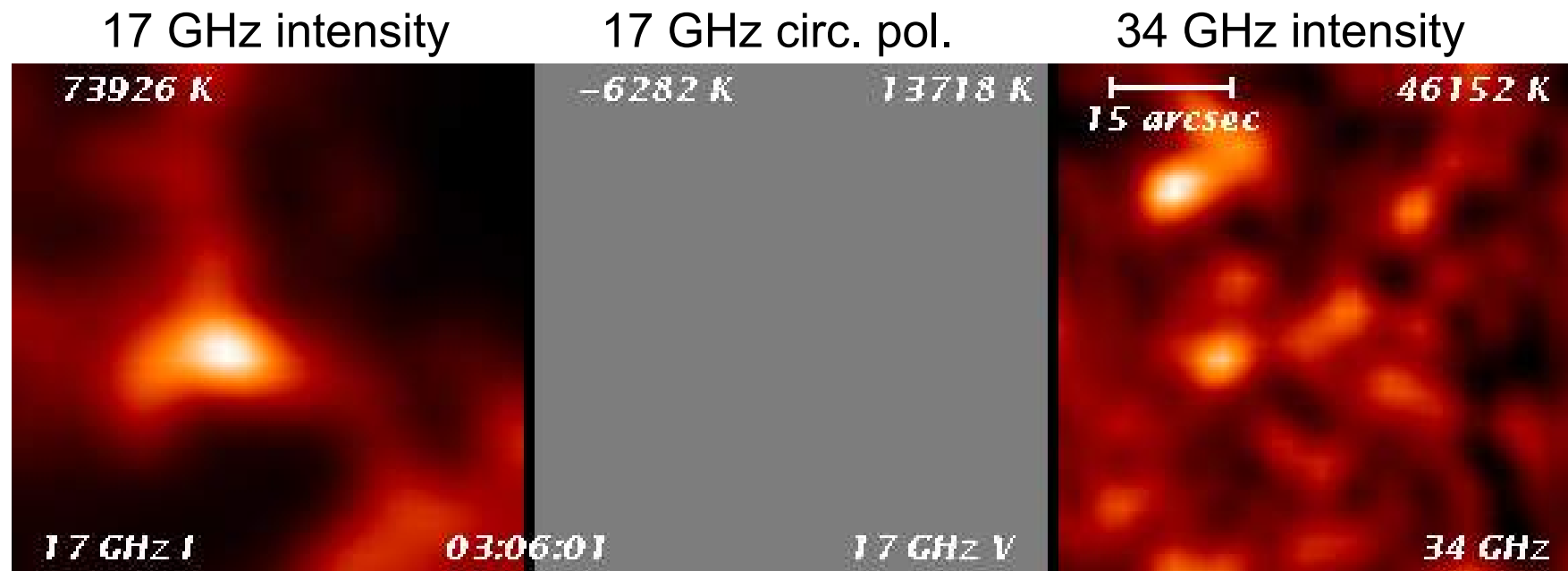




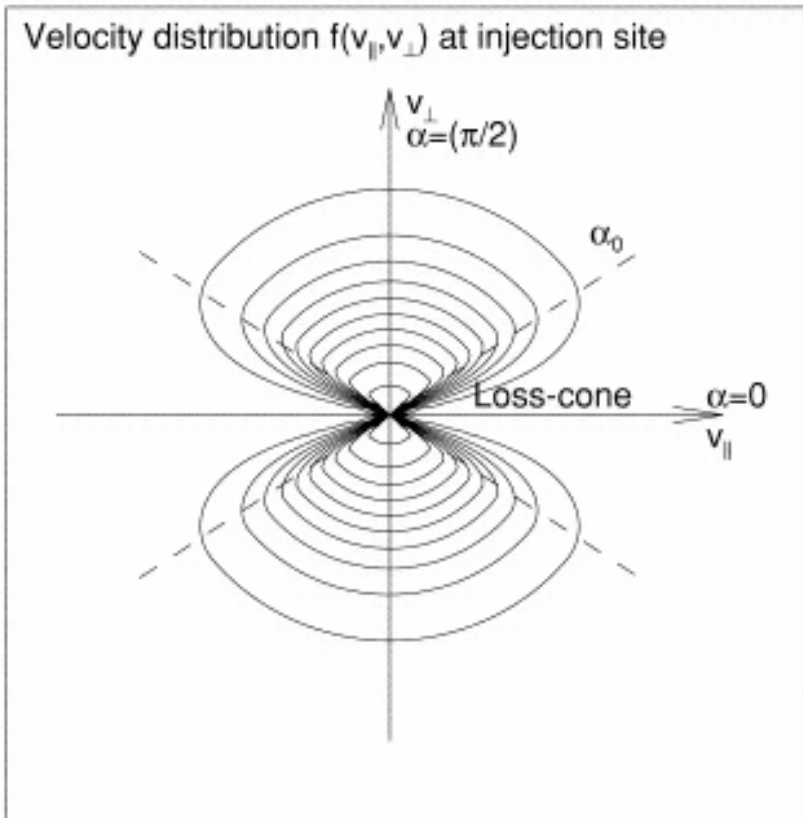
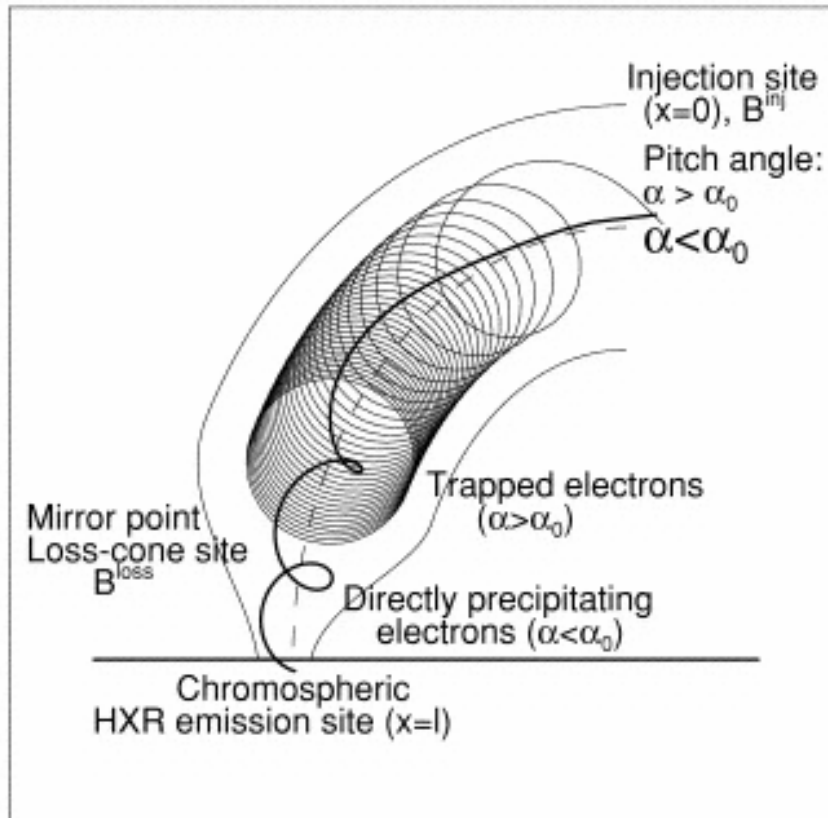
# 1999 August 28

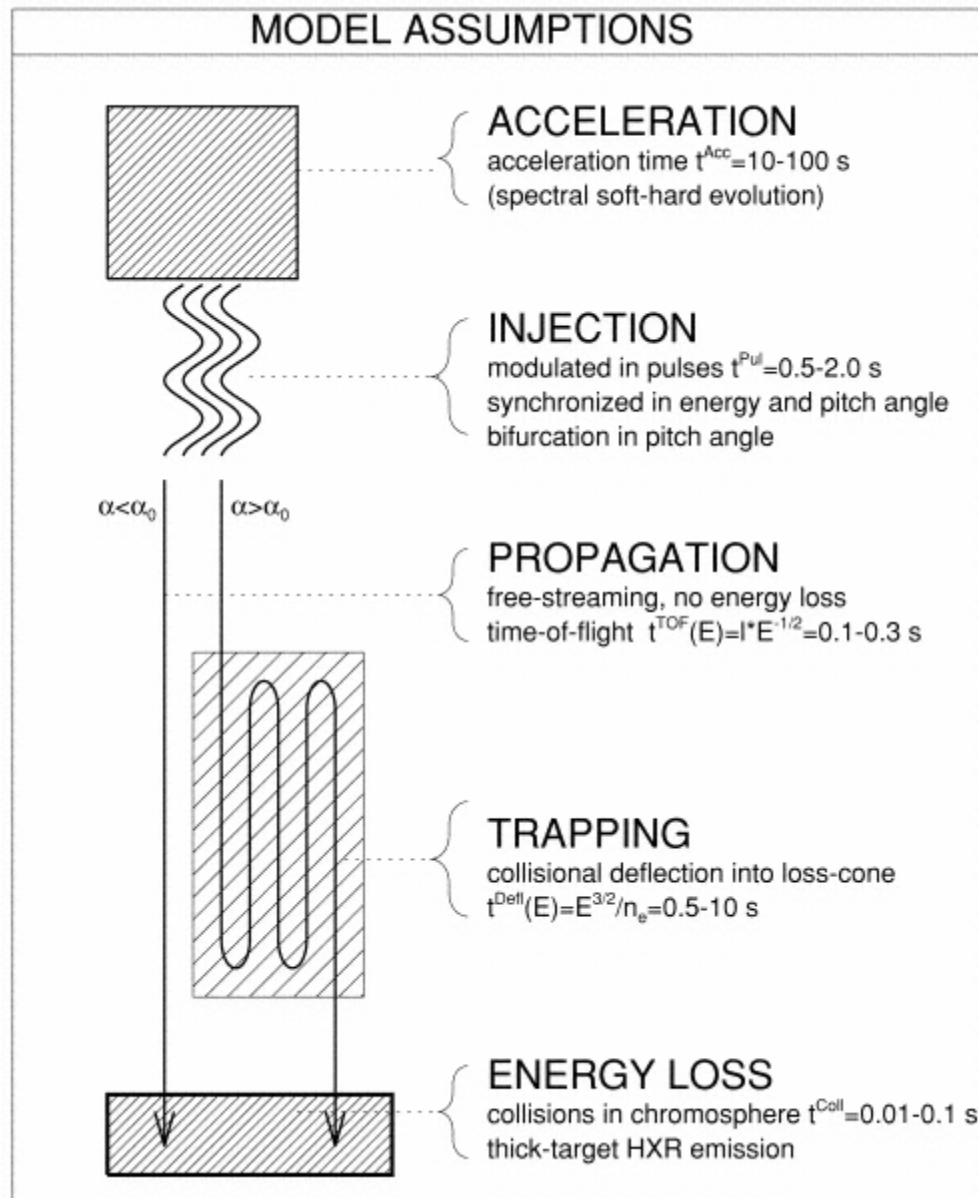






Magnetic field lines in the solar corona illuminated by gyrosynchrotron emission from nonthermal electrons.

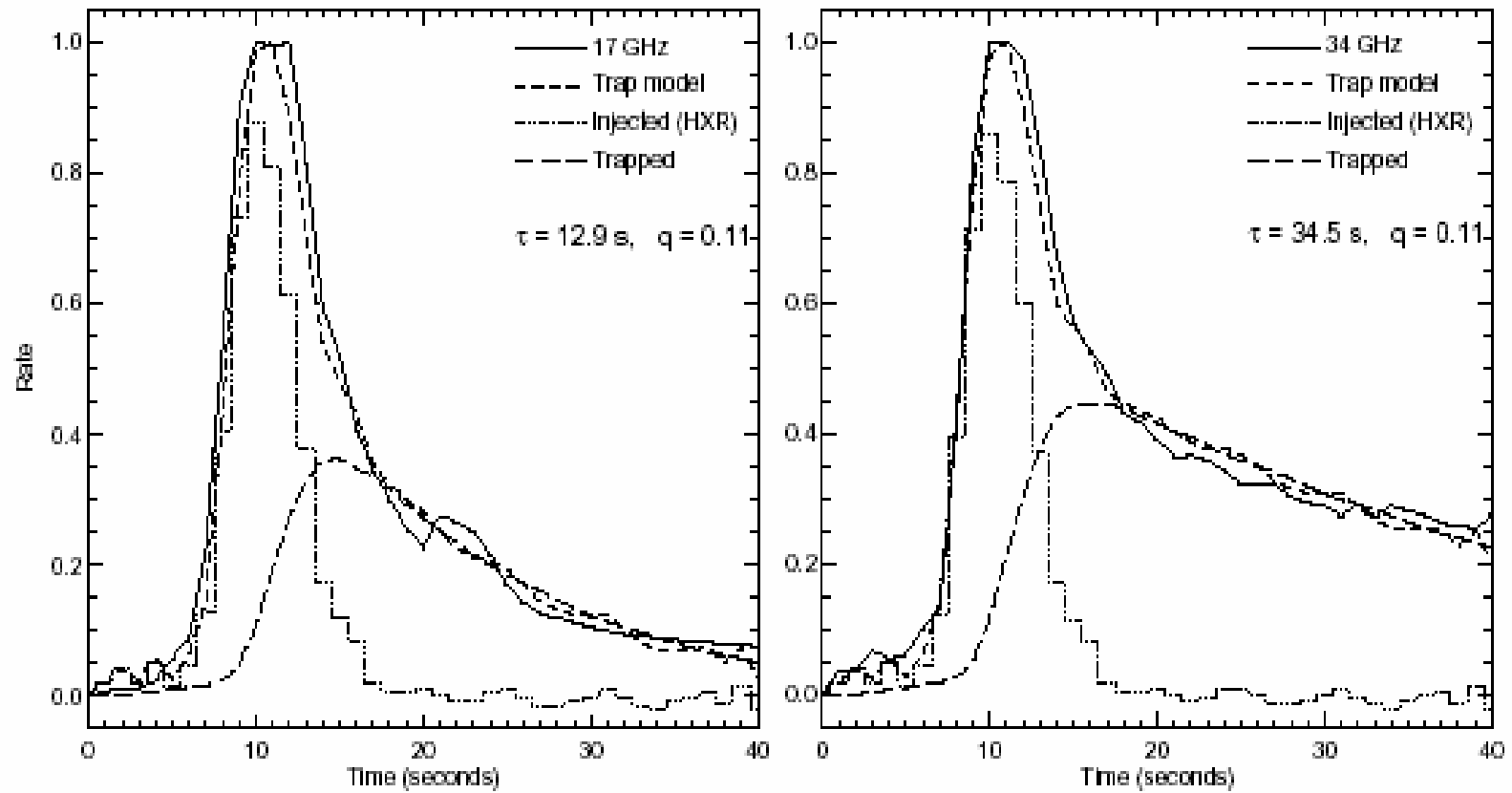




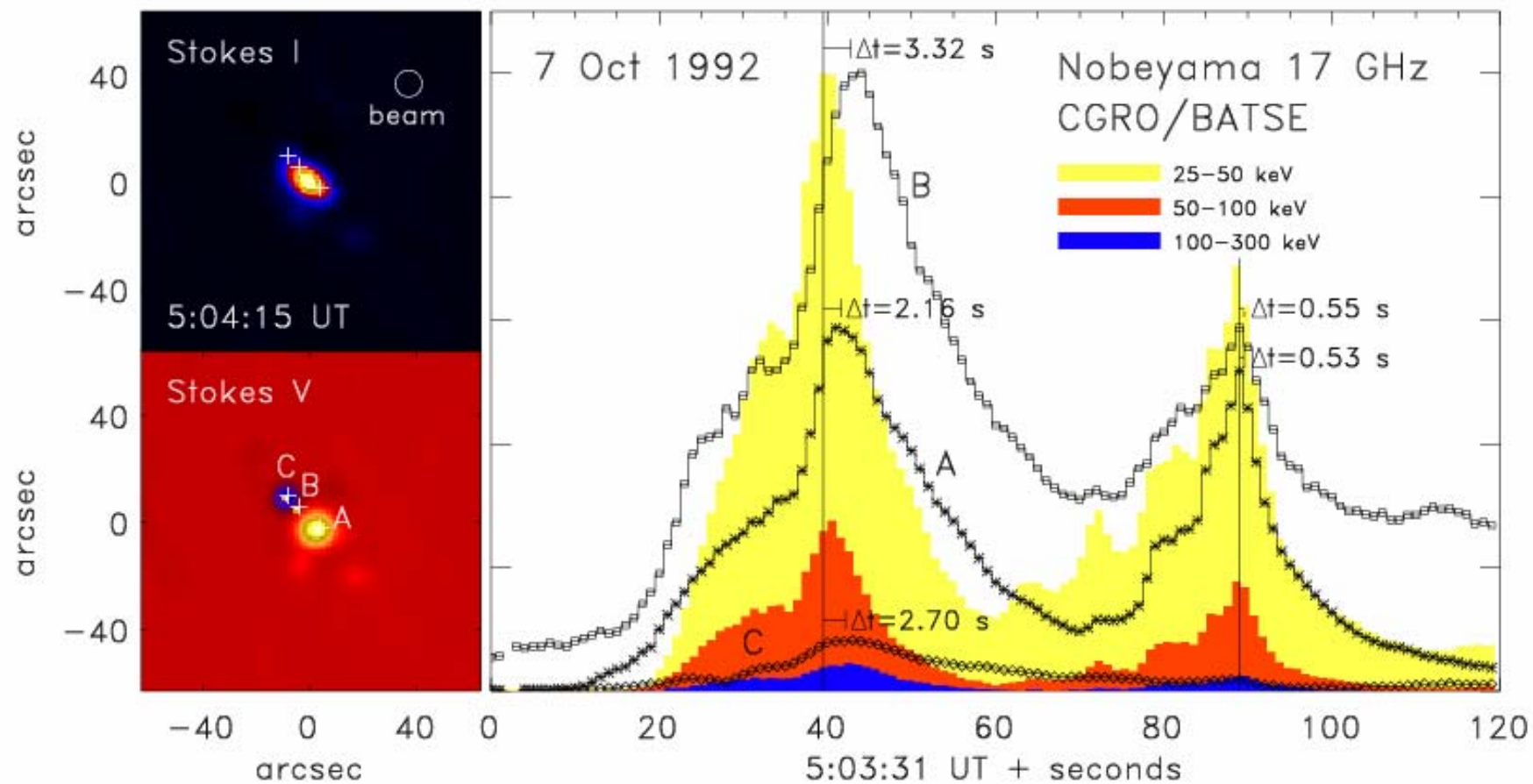
## TPP/DP Model

$$v_D \propto v_E \propto E^{-3/2}$$

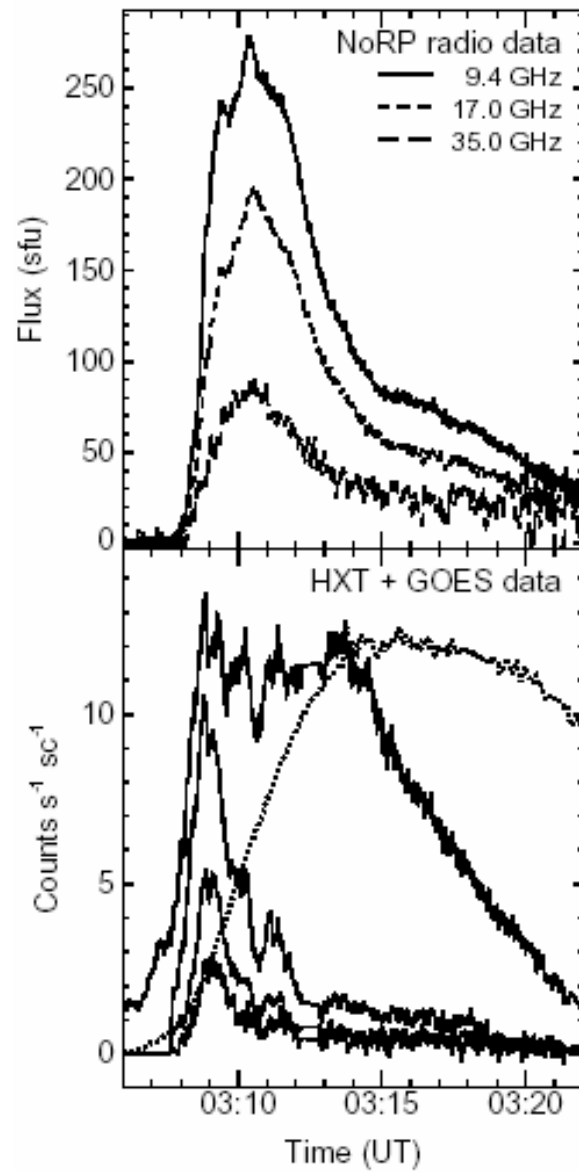
# 1998 June 13



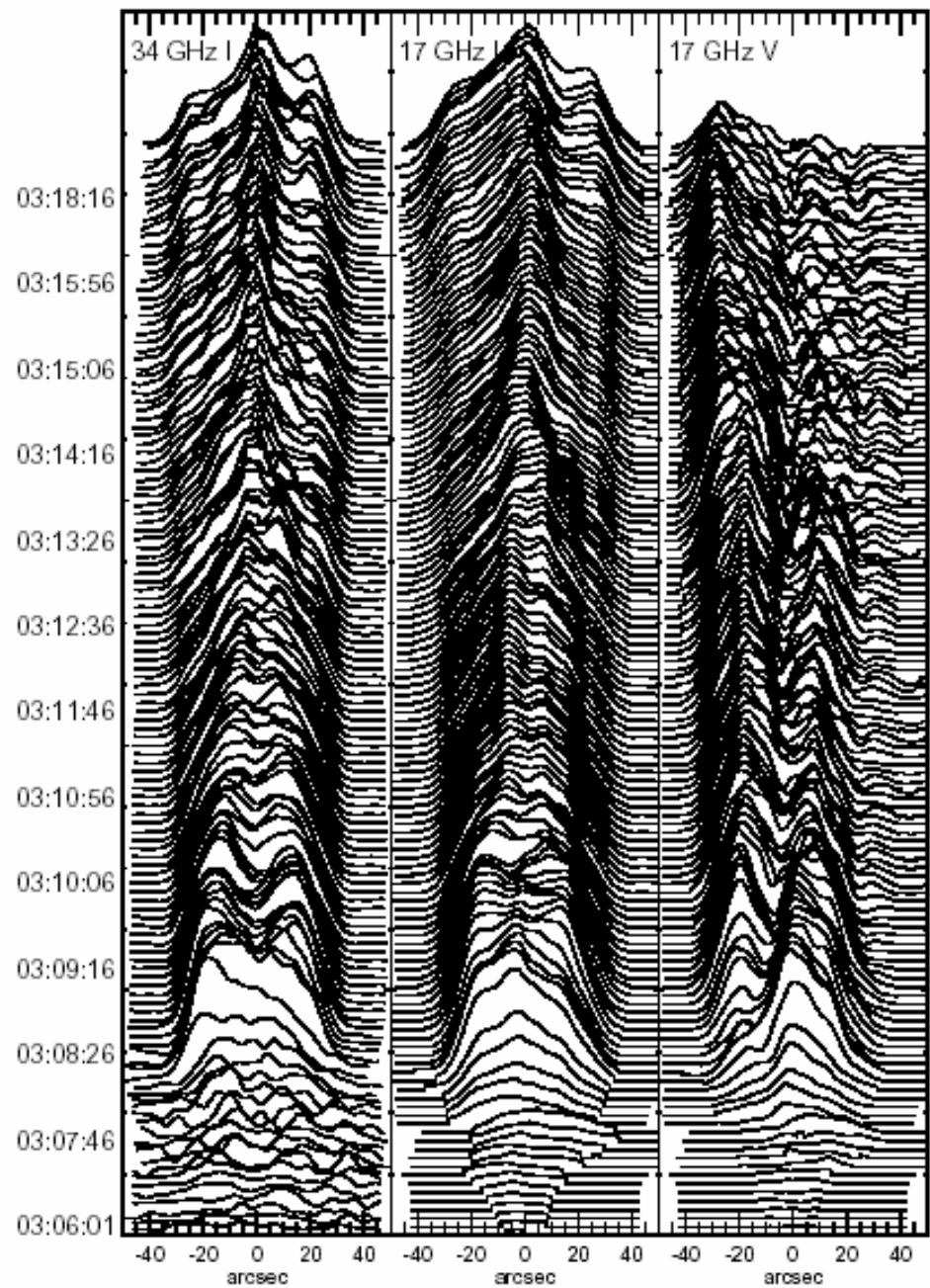
Kundu et al. 2001







1999 May 29

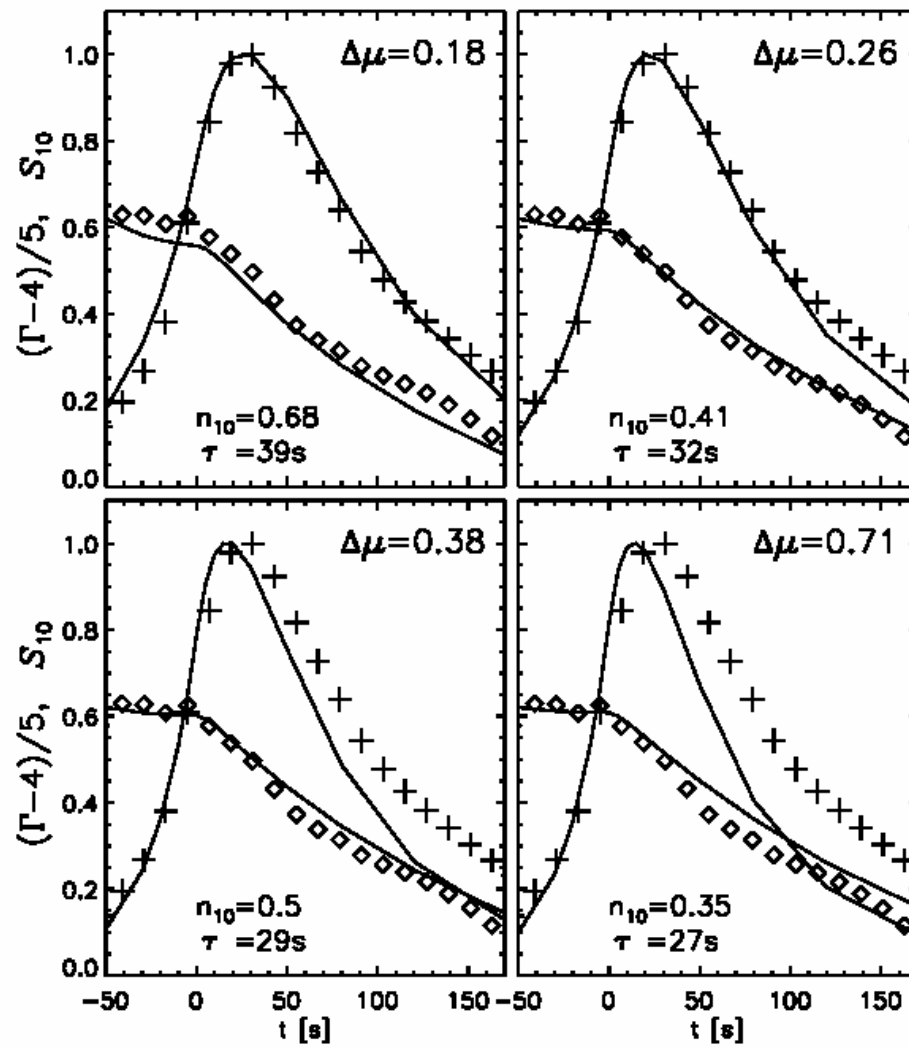
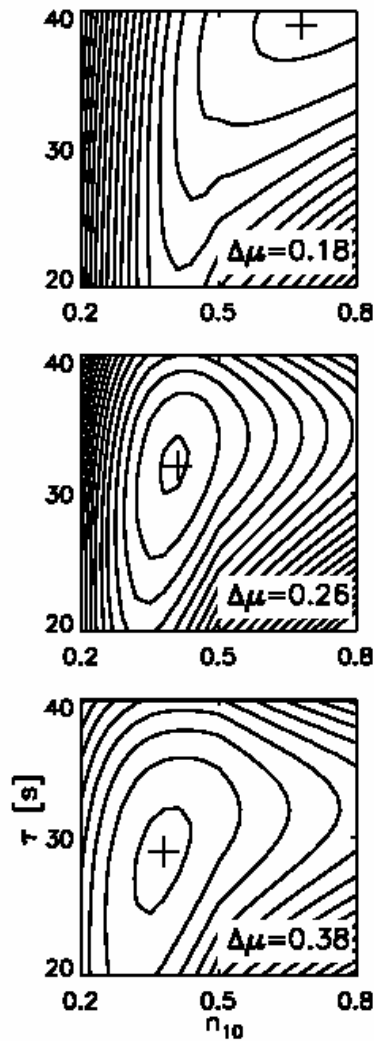


White et al 2002



1993 June 3

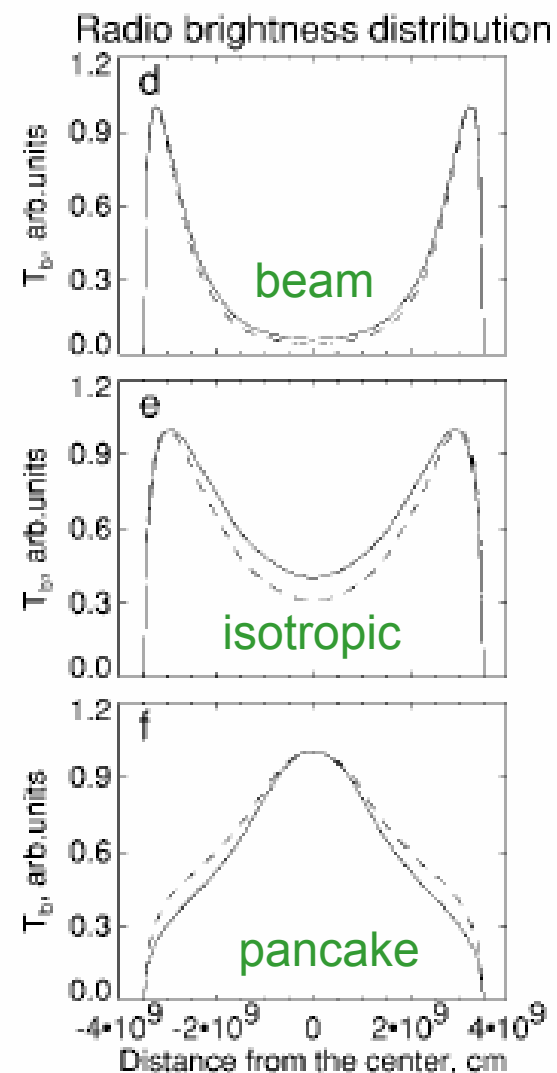
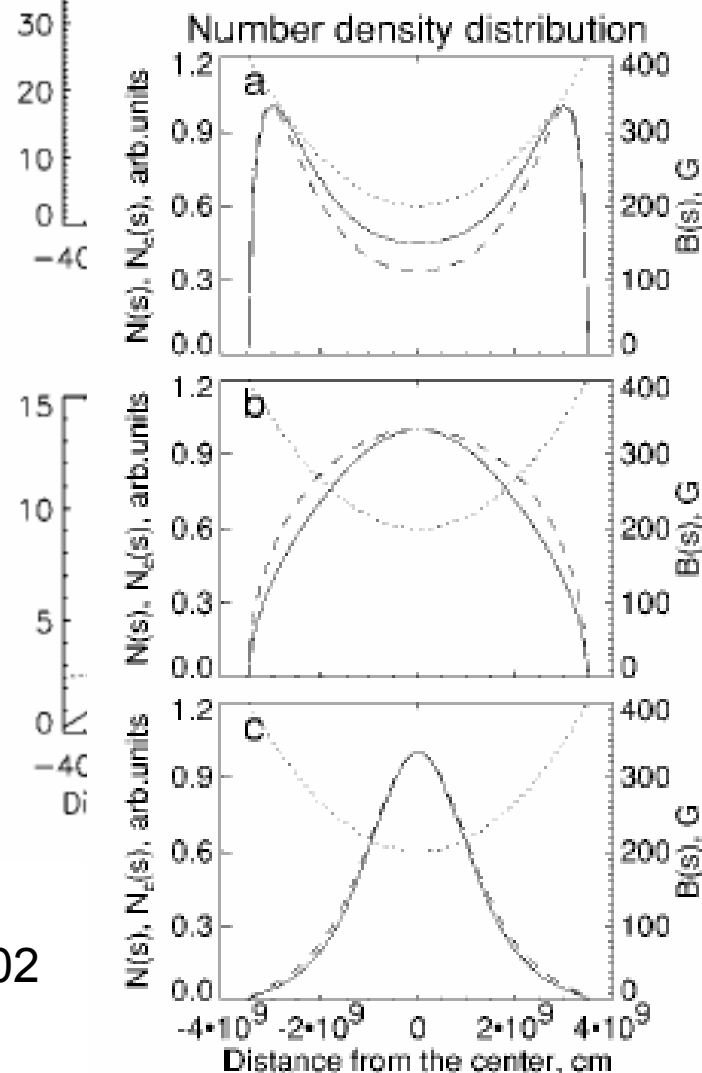
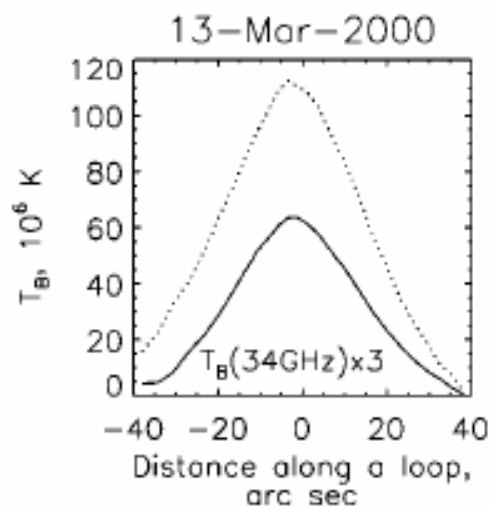
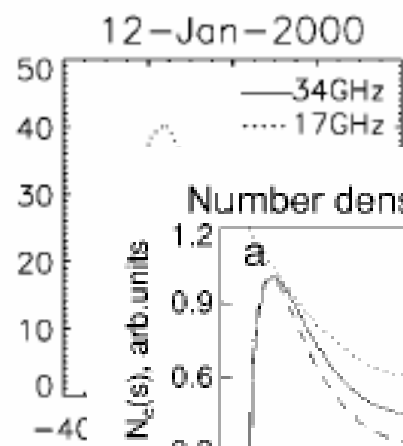
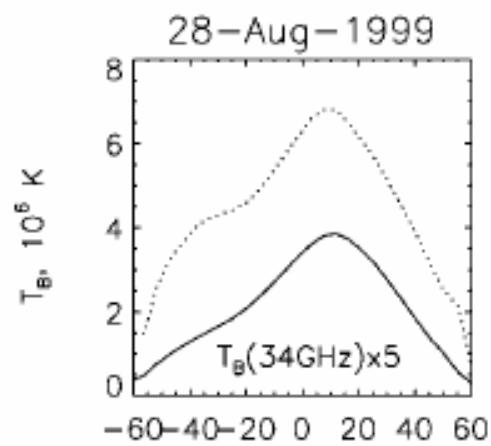
OVSA



$\mu_o=0$

$$P_\gamma(\mu)|_{s=0} = \frac{1}{\sqrt{\pi\Delta\mu}} \exp \left[ -4 \frac{(\mu - \mu_0)^2}{\Delta\mu^2} \right].$$

Lee & Gary 2000



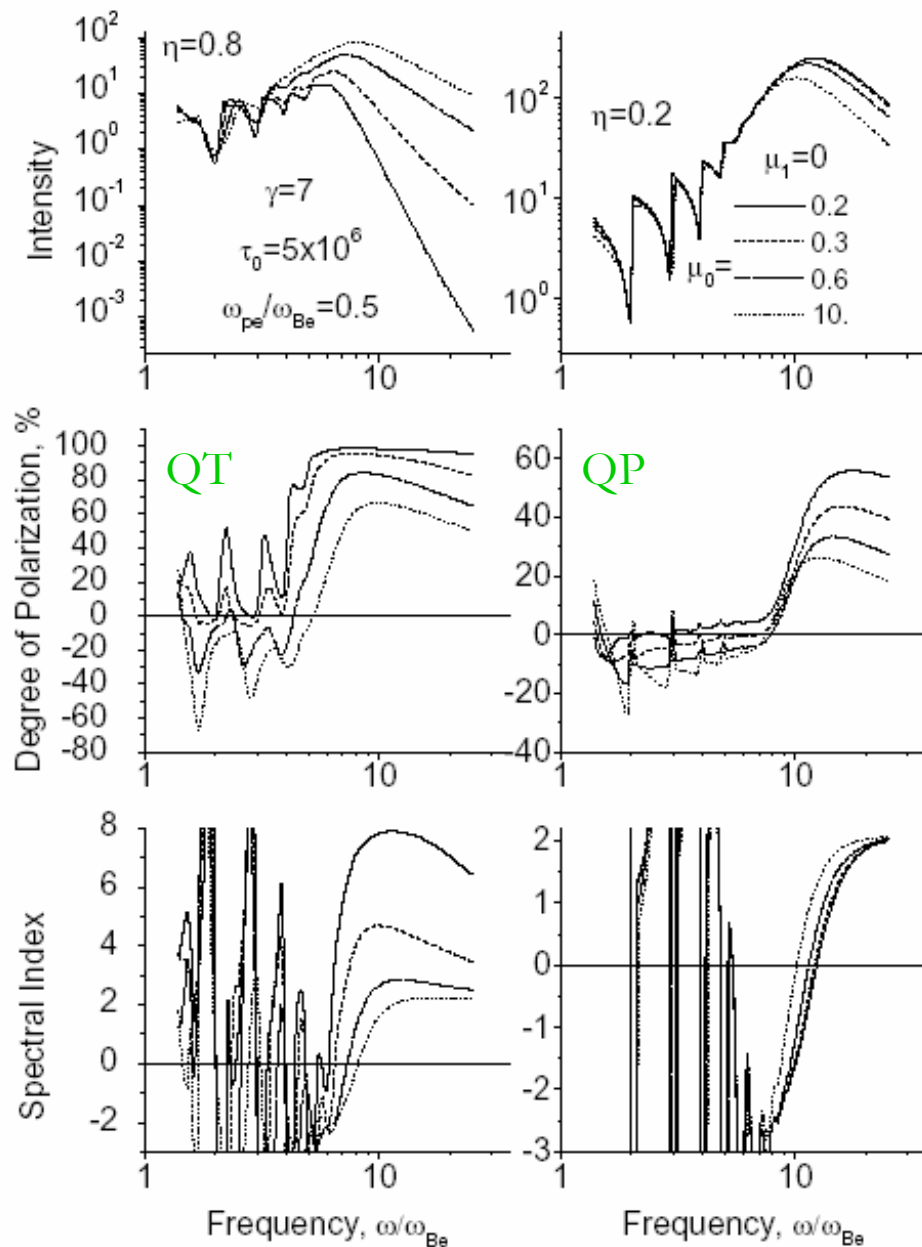
Melnikov et al 2002

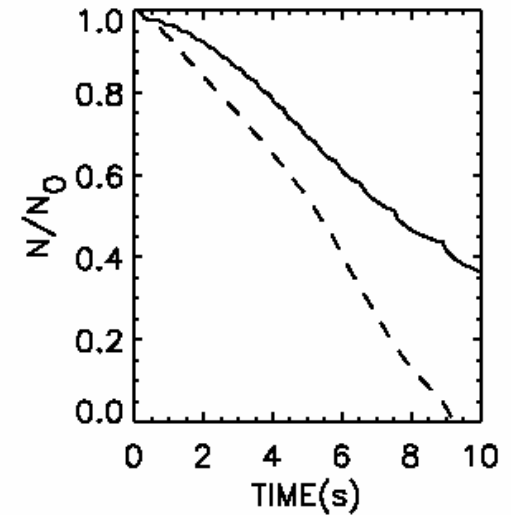
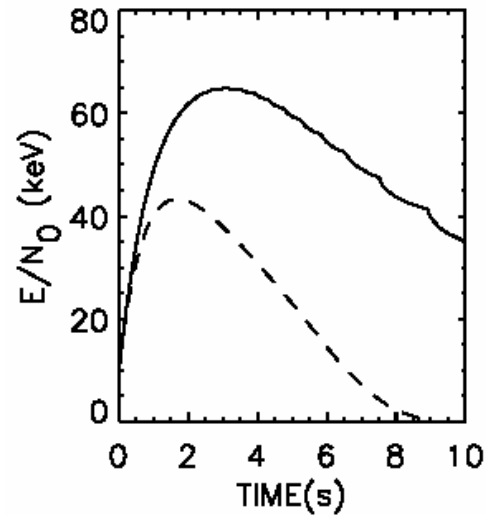
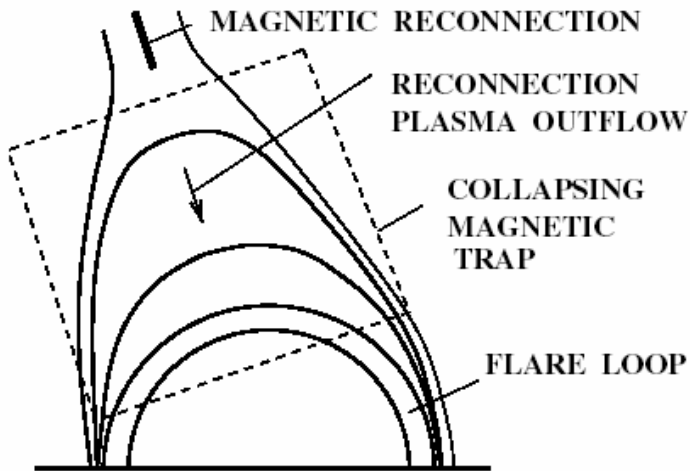
# Gyrosynchrotron radiation from anisotropic electrons

Fleishman & Melnikov 2003a,b

Properties of the emitted  
radiation – e.g., intensity,  
optically thin spectral index,  
degree of polarization –  
depend sensitively on the  
type and degree of electron  
anisotropy

$$\eta = \cos \theta$$

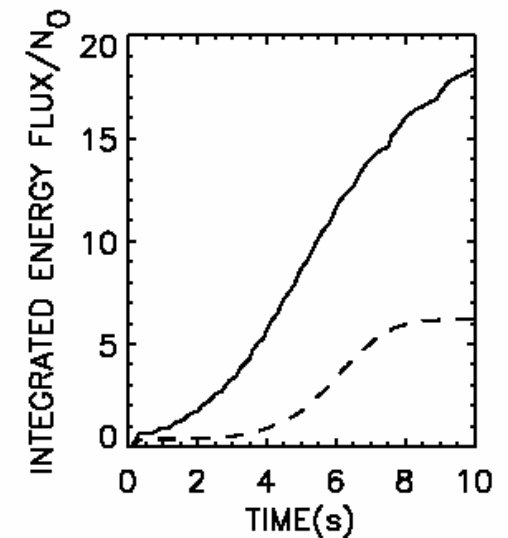
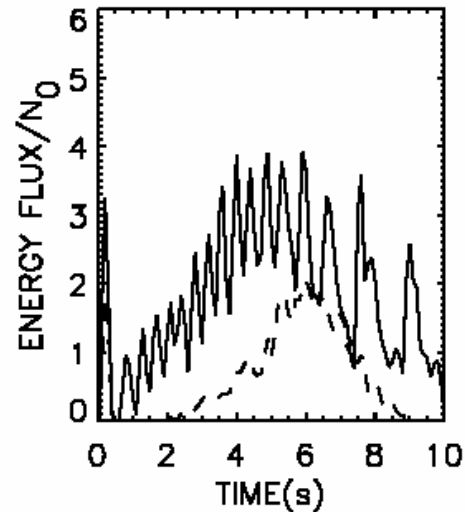




## “Collapsing trap”

Karlický & Kosugi 2004

- Analysis of betatron acceleration of electrons due to relaxation of post-reconnection magnetic field lines.
- Energies electrons and producers highly anisotropic distribution

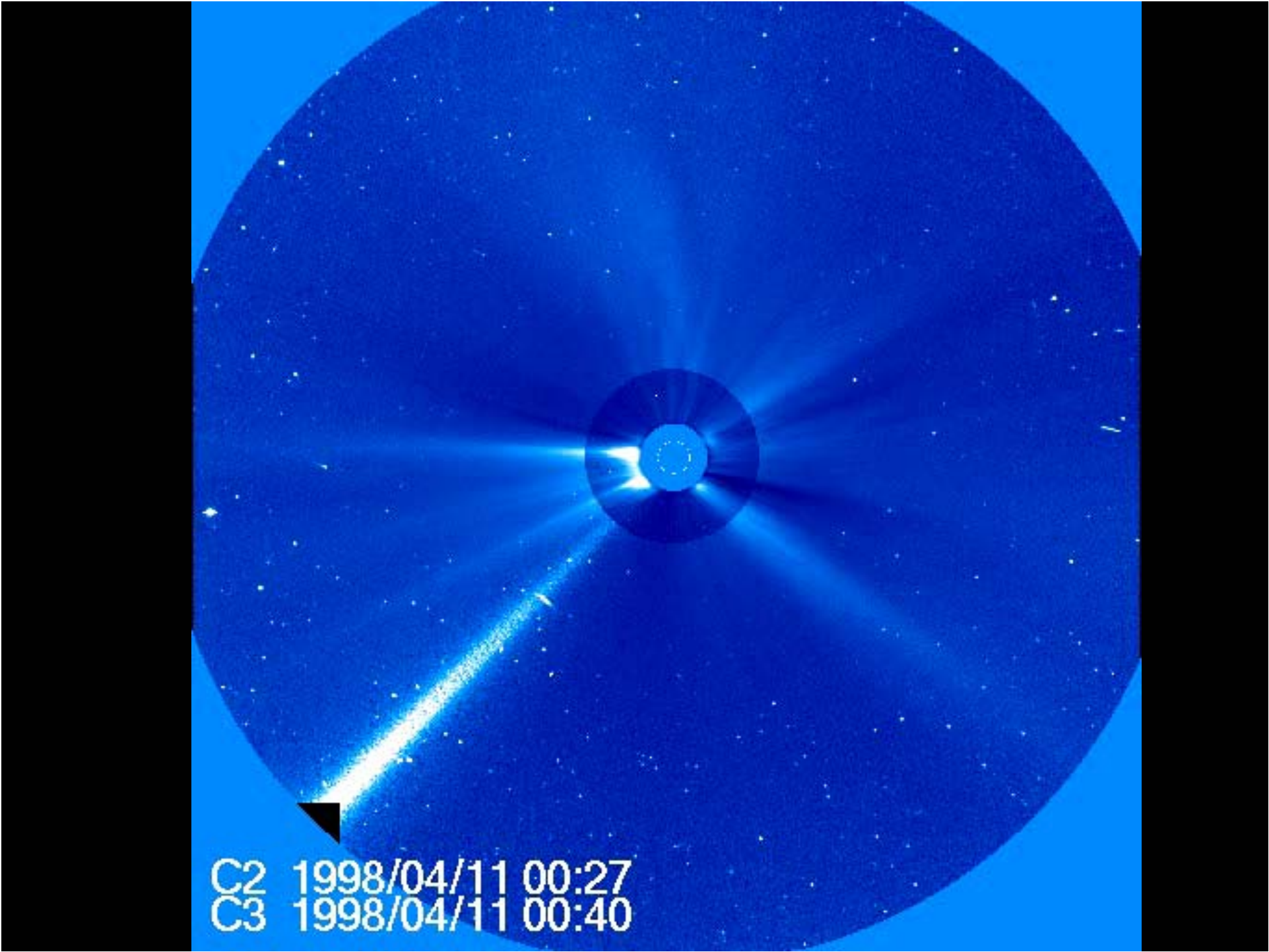


Model electron properties near “footpoint”

# Observations of Radio emission from flares

- Access to nonthermal electrons throughout flaring volume: magnetic connectivity
- Sensitive to electron distribution function and magnetic vector, ambient density and temperature
- Observations over past decade have clarified relation of microwave-emitting electrons to HXR-emitting electrons: FP, LT, spectral properties
- Recent work has emphasized importance of particle anisotropies

Need an instrument capable of time-resolved broadband imaging spectroscopy to fully exploit radio diagnostics!



C2 1998/04/11 00:27  
C3 1998/04/11 00:40

20 April 1998

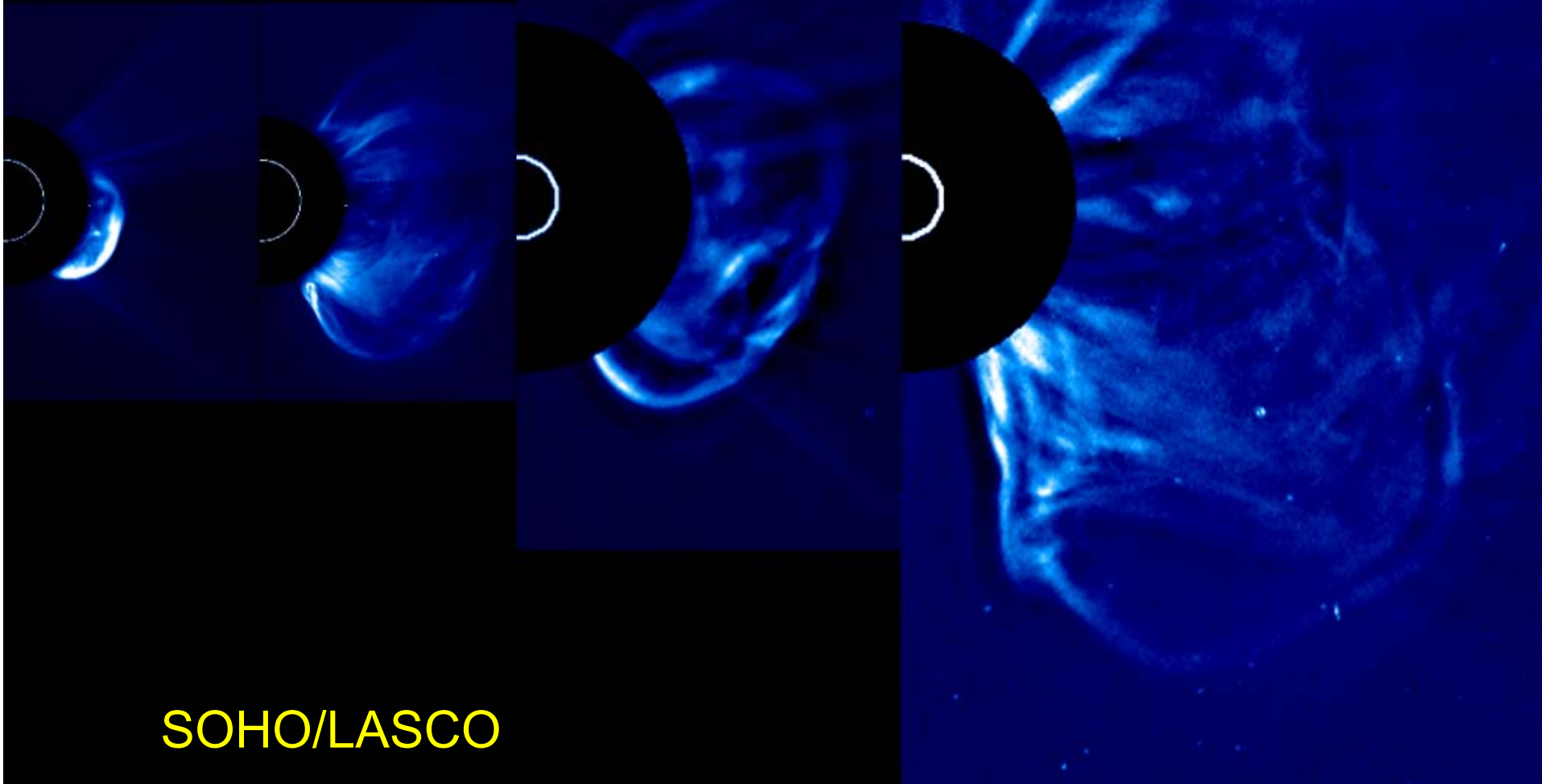
C2

C2

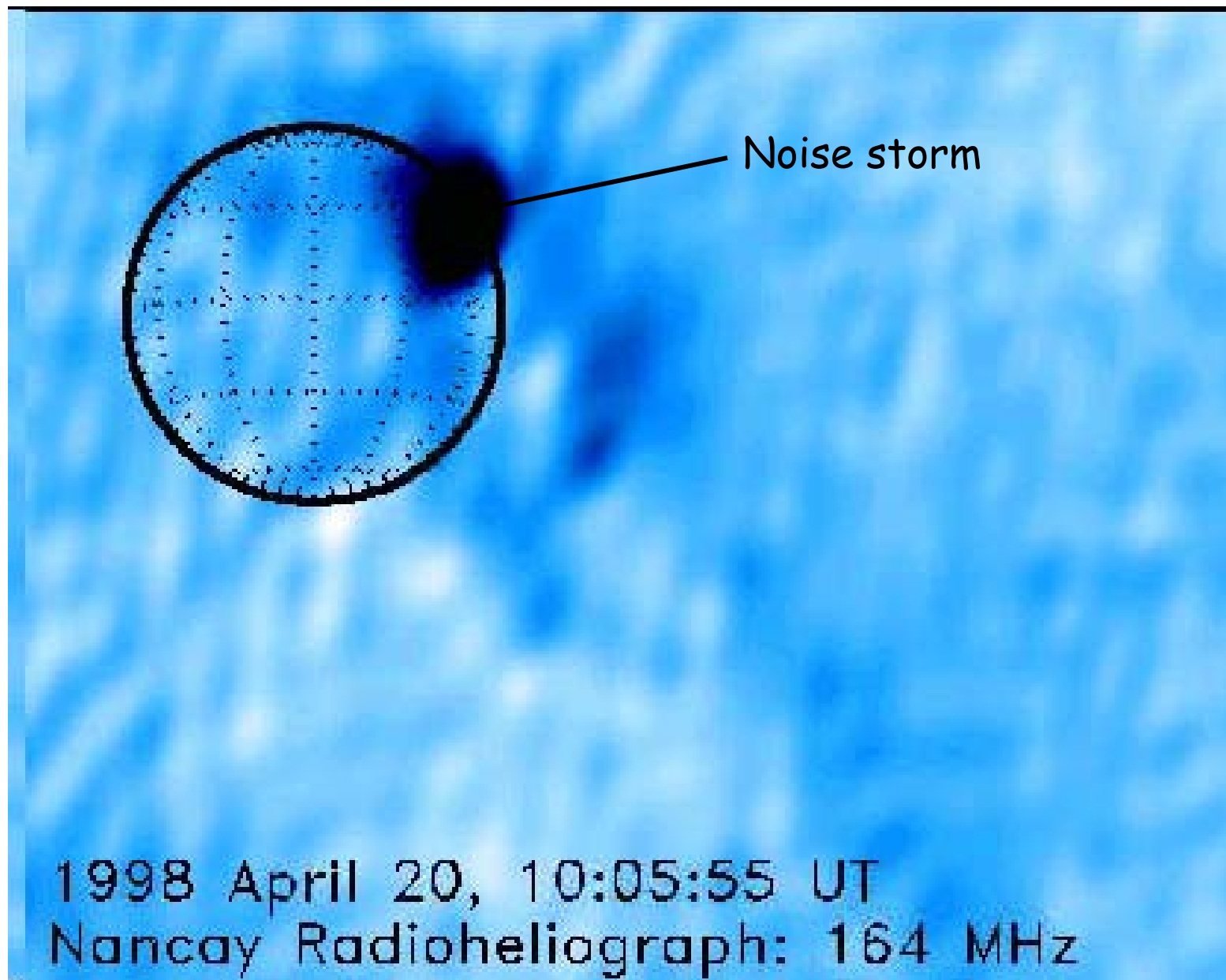
C3

C3

SOHO/LASCO

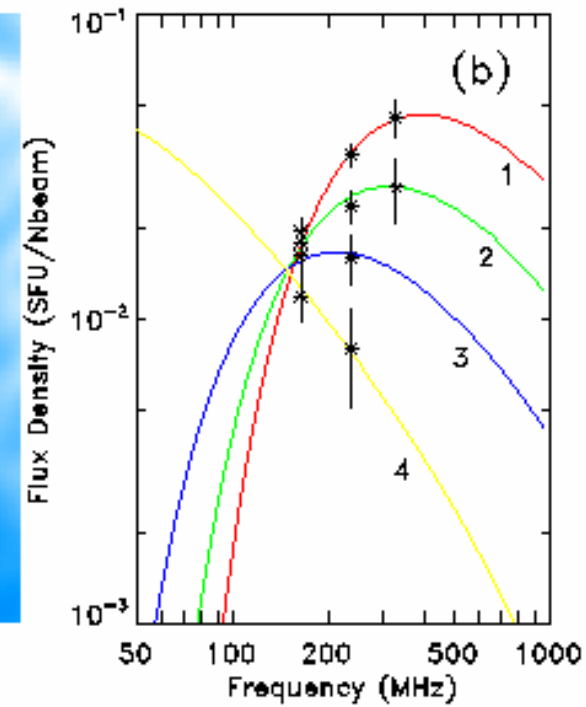
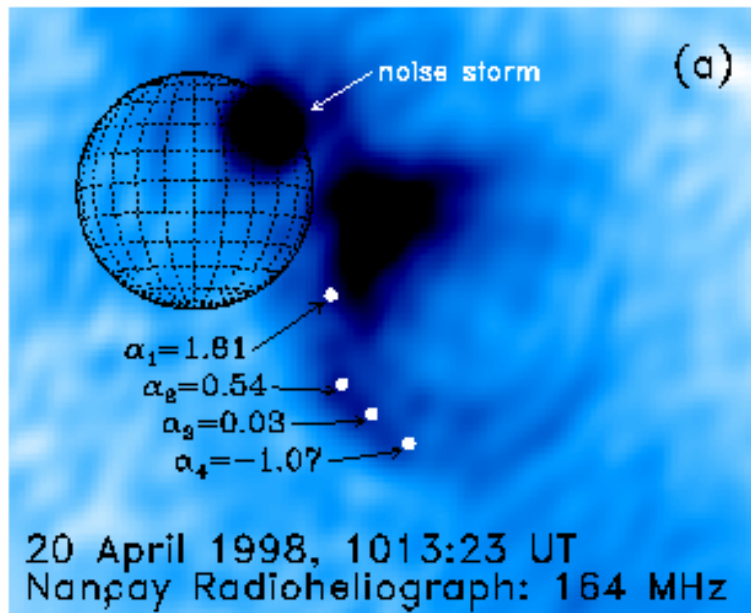






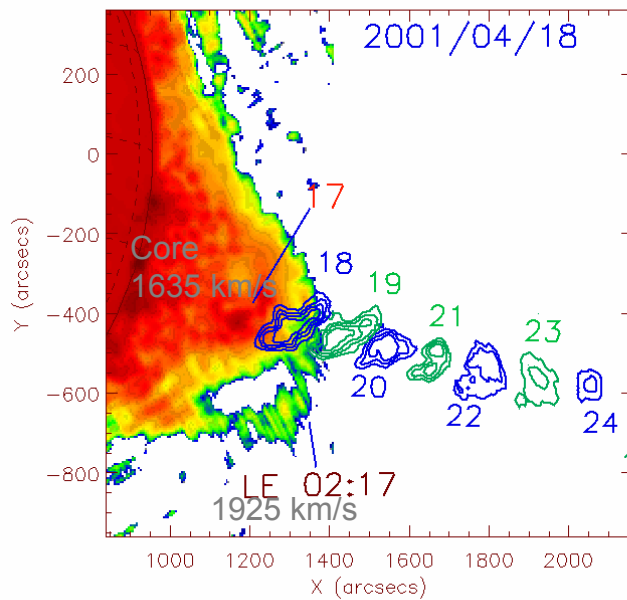
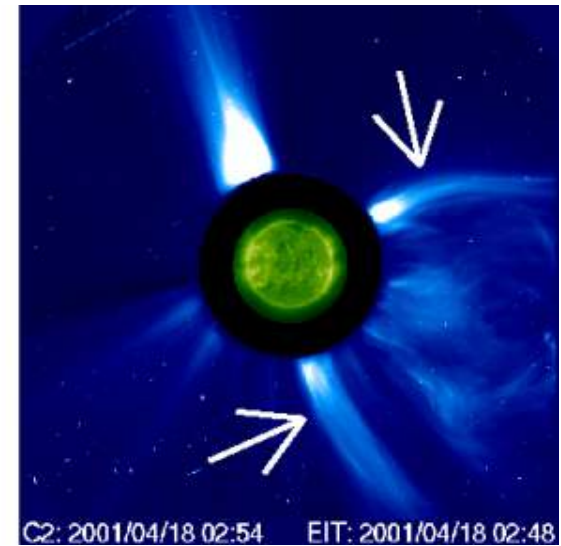
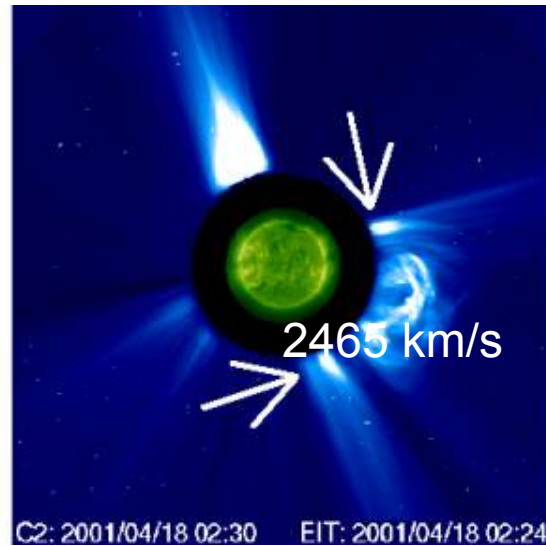
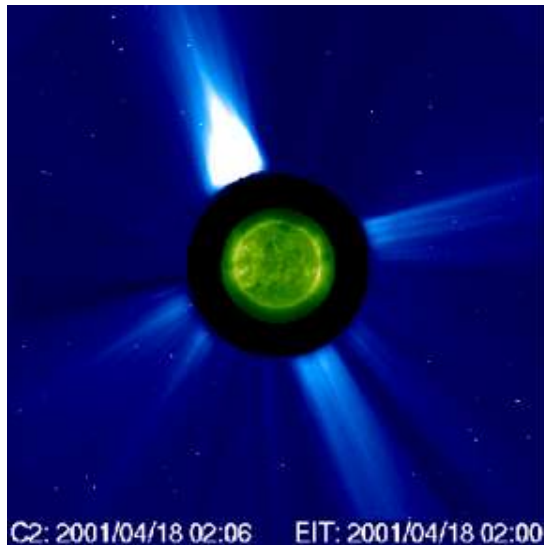
Bastian et al. (2001)



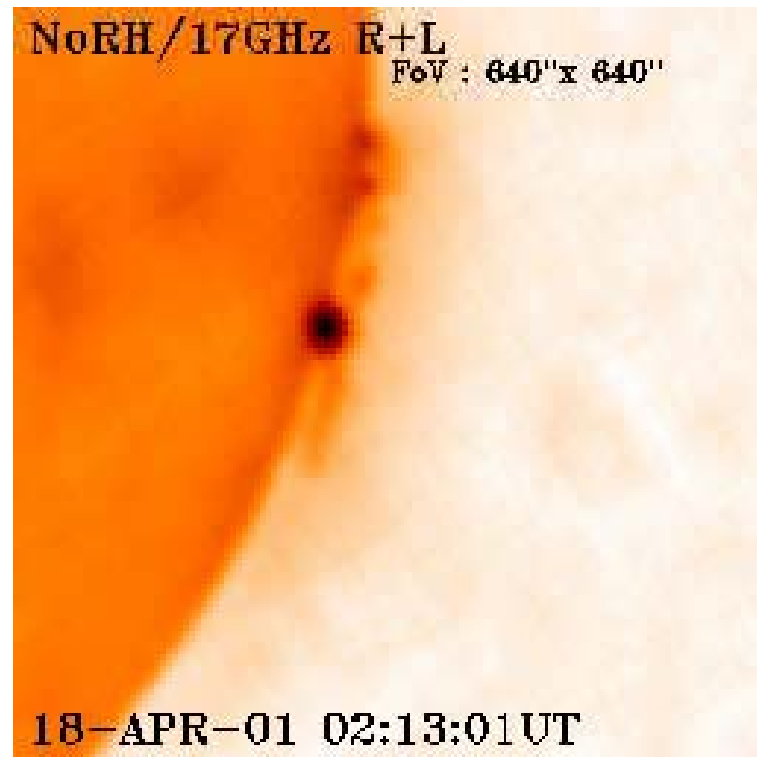


LoS	$\alpha$	$R_{\text{sun}}$	$\phi$ (deg)	$n_e$ (cm <sup>-3</sup> )	B(G)	$v_{\text{RT}}$ (MHz)
1	1.81	1.45	234	$2.5 \times 10^7$	1.47	330
2	0.54	2.05	218.5	$1.35 \times 10^7$	1.03	265
3	0.03	2.4	219.5	$6.5 \times 10^6$	0.69	190
4	-1.07	2.8	221	$5 \times 10^5$	0.33	30

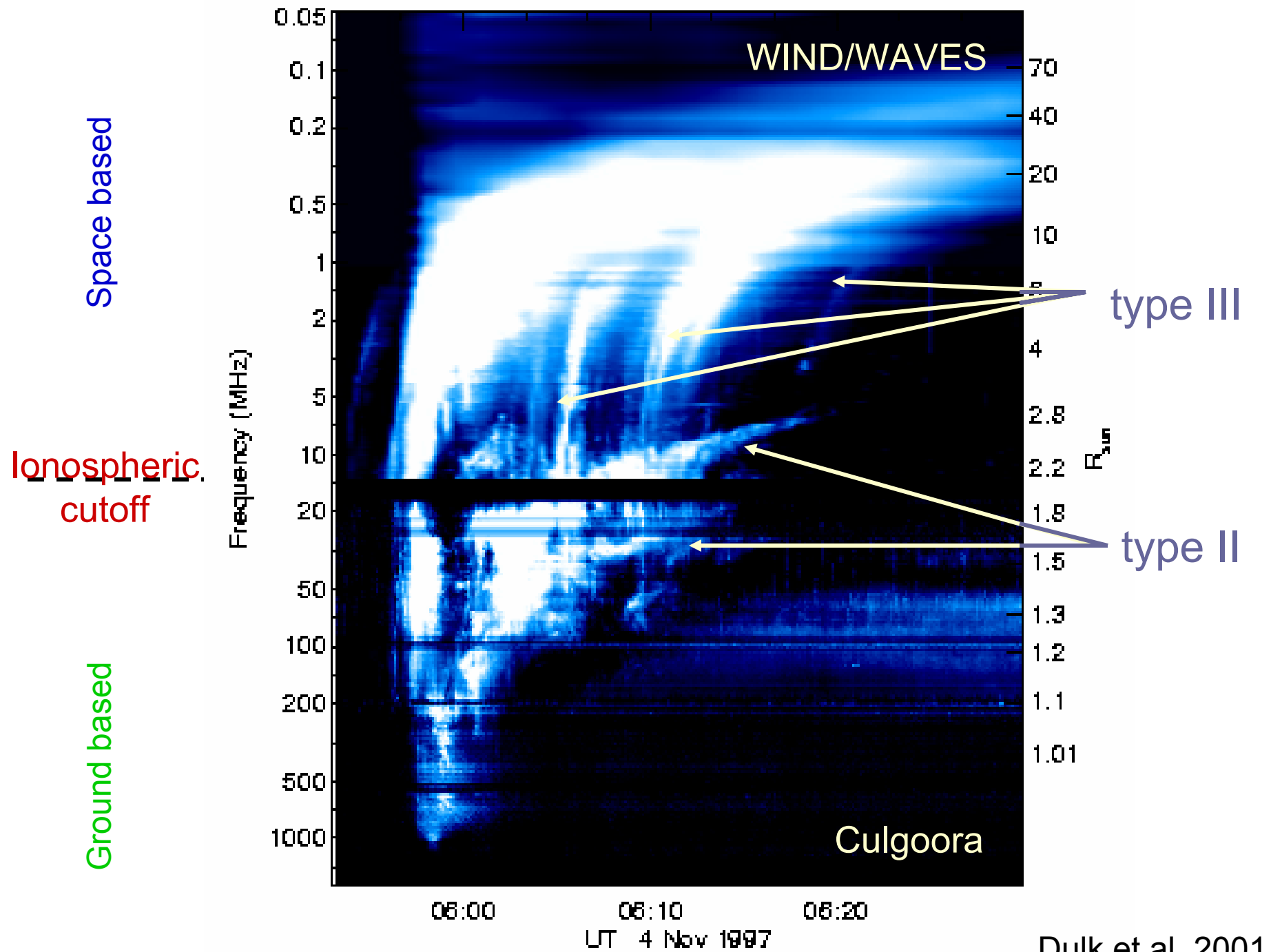
Bastian et al. (2001)



see Hudson et al. 2001



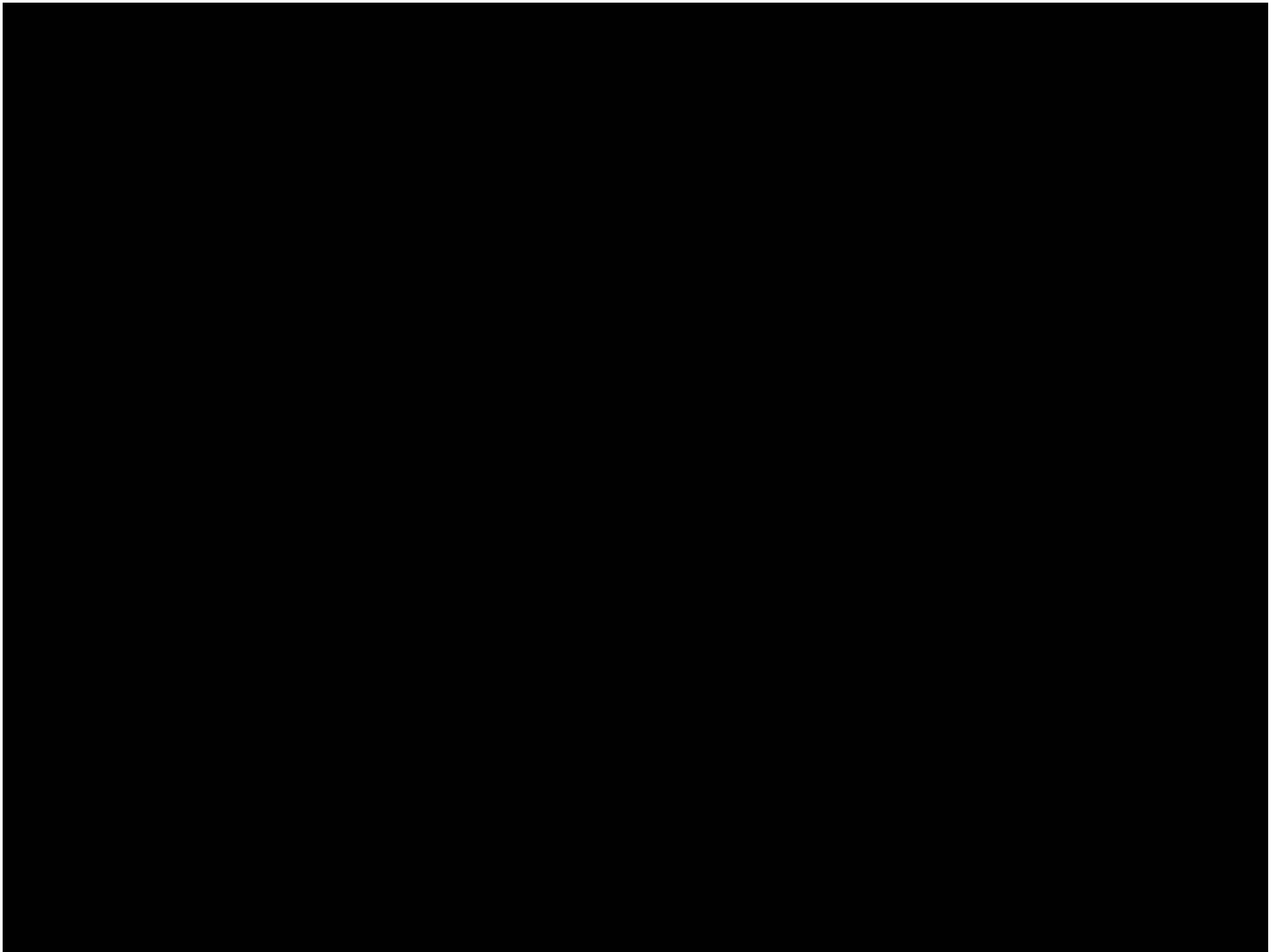
Gopalswamy et al 2004



# Observations of Radio emission from CMEs

- Unique access to the nascent stages of CMEs
- Sensitive to both gyrosynchrotron (leading edge) and thermal (core) emission
- Provides means of measuring speed, acceleration, width etc.
- Can also measure  $B$  (CME),  $n_{th}$  (CME),  $n_{rel}$  (CME),  $n_{th}$  (core),  $T$  (core)

Need an instrument capable of time-resolved  
broadband imaging spectroscopy to fully exploit radio  
diagnostics!



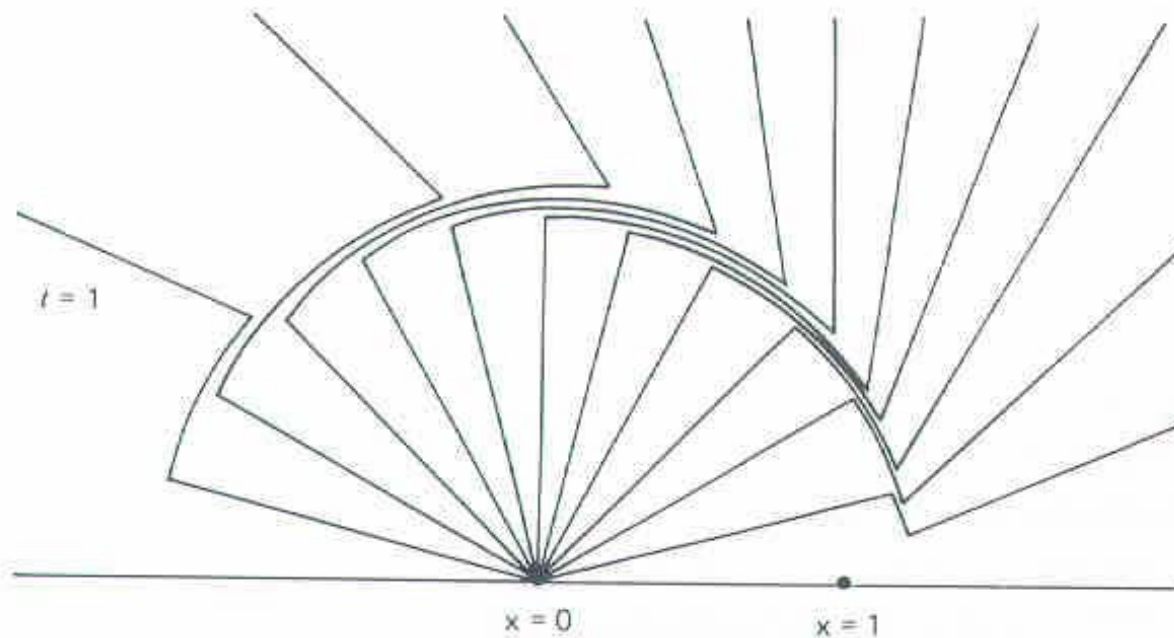
# Larmor Formula

For the time being, we are going to consider continuum emission mechanisms, deferring emission and absorption in spectral lines until later.

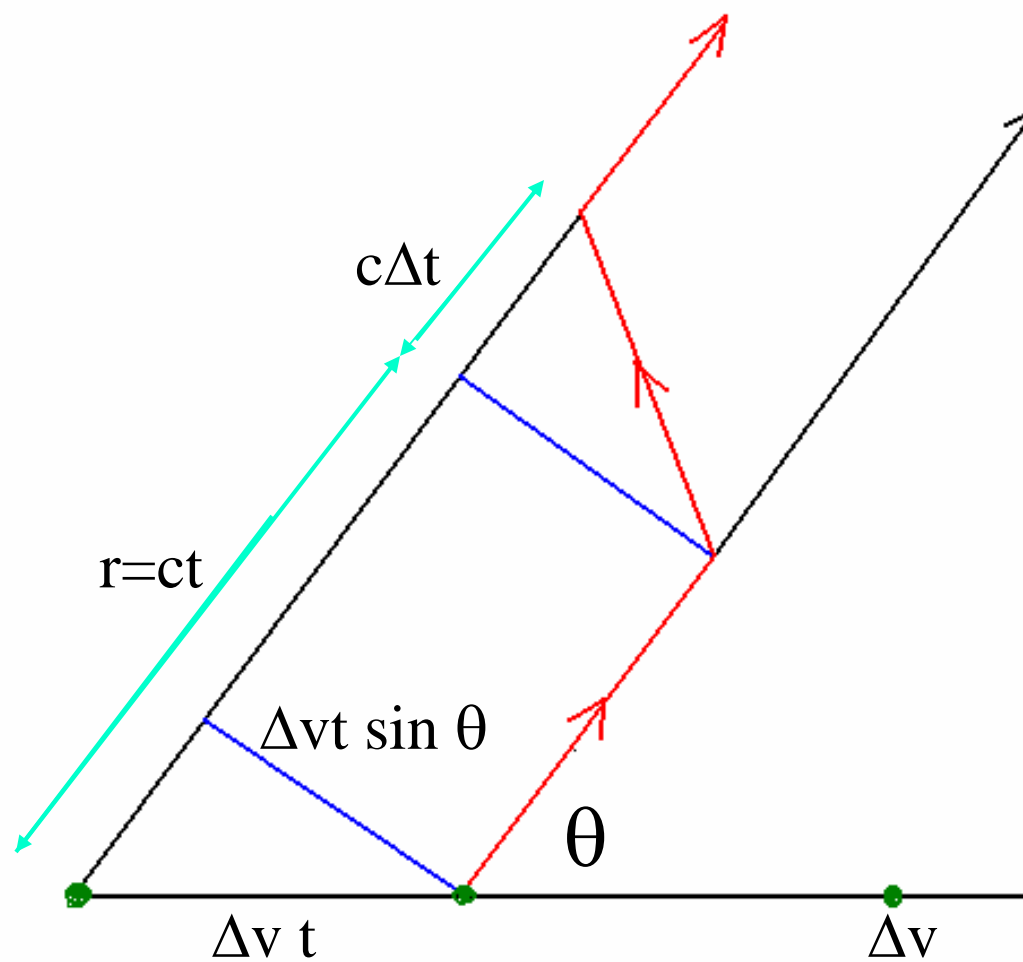
We are therefore going to ignore radiation processes involving atomic and molecular transitions and instead think about radiation from free charges.

A derivation of radiation from free charges is somewhat involved. I'm just going to sketch out the underlying physical ideas using a simple derivation due to [J.J Thomson](#).

First, consider a charged particle  $q$  moving at some velocity  $v$  from left to right. It is suddenly brought to a stop at point  $x$  at time  $t=0$ ; i.e., it is decelerated.



Alternatively, a charge can be accelerated. We'll analyze this case...





The ratio of the perpendicular component of the electric field to the radial component is

$$\frac{E_{\theta}}{E_r} = \frac{\Delta v \, t \sin \theta}{c \Delta t}$$

$$E_r = \frac{q}{r^2}$$

from Coulomb's Law. Since  $r=ct$ , we can substitute and rearrange terms to get

$$E_{\theta} = \frac{q}{rct} \frac{\Delta v}{\Delta t} \frac{t \sin \theta}{c} = \frac{q \sin \theta}{rc^2}$$

Note that  $E_r$  is proportional to  $1/r^2$ , while  $E_\theta$  is proportional to  $1/r$ , so  $E_\theta \gg E_r$  far from the charge  $q$ .

The radiation power is given by the Poynting Flux (power per unit area:  $\text{ergs cm}^{-2} \text{ s}^{-1}$  or  $\text{watts m}^{-2}$ )

$$\mathbf{S} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{H} = \frac{c}{4\pi} E^2$$

Substituting our expression for  $E_\theta$  for  $E$ , we obtain

$$|\mathbf{S}| = \frac{c}{4\pi} \left( \frac{q \sin \theta}{rc^2} \right)^2 = \frac{q^2 \sin^2 \theta}{4\pi c^3 r^2}$$

The power pattern is determined by the factor  $\sin^2\theta$ , which yields a “donut” (dipole pattern) whose axis is coincident with the vector along which the charge was accelerated.

To compute the total radiation power  $P$  produced by the accelerated charge, we integrate over the sphere of radius  $r$ :

$$P = \iint S \, dA = \frac{q^2 \ddot{x}^2}{4\pi c^3} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{\sin^2 \theta}{r^2} r \, d\theta \, r \sin \theta \, d\theta$$

$$P = \frac{q^2 \ddot{x}^2}{2c^3} \int_{\theta=0}^{\pi} \sin^3 \theta \, d\theta = \frac{2}{3} \frac{q^2 \ddot{x}^2}{c^3}$$

The last expression is called Larmor's Equation. Note that the power is proportional to the square of the charge and the square of the acceleration.