Count Rates and Visibilities

Given an incident flux of photons on a HESSI subcollimator, after the selection of a map center, the predicted count rate $\mathcal{C}$ is given by the following expression:

$$\mathcal{C} = F_0 \; T \; \tau \;\; \cdot \{1 \; + \; c_1 \; cos[\Phi - \Phi_1] + c_2 \; cos[2(\Phi - \Phi_2)] \; + \; \cdots\} \qquad (1)$$

where

$$\begin{aligned} F_0 &= \text{Incident photon flux on front grid} \\ T &= \text{Subcollimator transmission} \\ \tau &= \text{Livetime} \\ \Phi &= \text{Phase at map center} \\ c_n &= \text{Modulation Amplitude,} \quad n = 1, 2, 3, 4, 5 \\ \Phi_n &= \text{Modulation Phase,} \quad n = 1, 2, 3, 4, 5 \end{aligned}$$

In general, the phase at map center depends on the results of the aspect solution, the time binning, the subcollimator, and the position of the map center. The phase at map center Φ depends on the the projection of the vector between the subcollimator axis (x_{coll}, y_{coll}) and the map center (x_{map}, y_{map}):

$$\Phi = 2\pi\{[x_{map} - x_{coll}]cos\theta + [y_{map} - y_{coll}]sin\theta)/p\} \qquad (2)$$

where

$$\begin{aligned} (x_{coll}, y_{coll}) &= \text{Subcollimator direction in inertial coordinates} \\ (x_{map}, y_{map}) &= \text{Map center in inertial coordinates} \\ \theta &= \text{Azimuth angle of slits} \\ p &= \text{Pitch of subcollimator grids} \end{aligned}$$

In the simplest case, where, the spin axis is at the origin and is stationary, the phase at map center is a function only of the azimuth ϕ, the pitch p, and the location of the map center (R_0, ϕ_0):

$$\Phi = 2\pi R_0 cos(\phi - \phi_0)/p \qquad (3)$$

But in general, regardless of the position of the spin axis, the predicted count rate is:

$$\mathcal{C} = F_0\ T\ \tau\ \cdot \{1\ +\ c_1\ cos\Phi\ +\ \cdots\} \tag{4}$$

The visibility of a point source at the map center is:

$$V = F_0\ e^{\mathbf{i}2\pi[x_{map}\ cos\theta + y_{map}\ sin\theta)]/p} \tag{5}$$

Note that the visibility does not depend on the subcollimator coordinates (x_{coll}, y_{coll}), but the count rate is subject to phase shifts caused by telescope motion. The phase shifts due to this motion are:

$$\Delta\Phi = 2\pi\{x_{coll}\ cos\theta + y_{coll}\ sin\theta)/p \tag{6}$$

And these are reflected in additional variations of the count rate profile. (The quantity $\Delta\Phi$ is computed from the aspect solution.) We can separate out this variation explicitly in the visibility:

$$V = F_0\ e^{\mathbf{i}[\Phi + \Delta\Phi]} \tag{7}$$

If we subtract out the mean of the count rate (equation 4), and divide out the $T\tau$ factor, keeping only the fundamental term,

$$(\mathcal{C} - F_0\ T\ \tau)/(T\ \tau c_1) = F_0\ cos\Phi \tag{8}$$

So, for this particular case of a point source at map center the relation between the countrate profile and the visibilities are found by comparing equations 7 and 8:

$$Re\{Ve^{-\mathbf{i}\Delta\Phi}\} = \frac{\mathcal{C} - F_0 T\tau}{c_1 T\tau} = \frac{\mathcal{C} - <\mathcal{C}>}{c_1 T\tau} \tag{9}$$

It is important to note that, by equation 9, the peak-to-peak variation of the visibility is several times larger than the peak-to-peak variation of $\mathcal{C}$. This is because the visibility reflects the transform of the incident photons, while the count rate is the modulated profile of only the detected photons. In fact, when the livetime is unity ($T = 1$), and when the subcollimator pattern is perfectly triangular, ($c_1 = 8/\pi^2$ and $\tau = 1/4$), the peak-to-peak variation of $Re\{V\}$ is $\pi^2/2 = 4.9$ times that of the count profile.

For a more general case, where the source is an arbitrary superposition of N point sources of flux F_j and location (R_j, ϕ_j), the visibility is:

$$V = e^{\mathbf{i}\Delta\Phi} \sum_{j=1}^{N} F_j\ e^{\mathbf{i}\Phi_j} \tag{10}$$

where the Φ_j are the phases of the point sources at (R_j, ϕ_j).
Then, the corresponding predicted count rate is:

$$\mathcal{C} = T\ \tau\ \cdot \sum_{j=1}^{N} F_j\ \{1\ +\ c_1\ cos[\Phi_j)]\ +\ \cdots\} \tag{11}$$

Comparing equations 10 and 11, we find that equation 9 remains valid, providing that F_0 is defined as the total flux of the N sources.

The generalization to higher harmonics is straightforward. The visibility for the n^{th} harmonic is given by:

$$Re\{V^{(n)} e^{-\mathbf{i}\Delta\Phi}\} = \frac{\mathcal{C}^{(n)}}{c_n T \tau} \tag{12}$$

where $\mathcal{C}^{(n)}$ is the n^{th} term in the Fourier expansion of the count rate (equation 1). Each visibility profile derived from equation 12 then defines a portion of the entire Fourier transform obtained on a circle of radius n/p in the Fourier plane.