

# **THICK-TARGET BREMSSTRAHLUNG**

## **CODE DOCUMENTATION**

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## Description

This program, BREMTHICK, computes the bremsstrahlung x-ray/gamma-ray spectrum from the interaction of energetic electrons with a thick-target plasma. The photon flux in photons  $\text{sec}^{-1} \text{keV}^{-1} \text{cm}^{-2}$  is computed as a function of photon energy. Angular dependence of the electron distribution function and radiation is not included. Therefore, both are assumed to be isotropic.

To run the program, the following files must be compiled and linked:

```
bremcross.for  
bremthick.for  
dmlinbr2.for  
fdblpbr.for
```

The IDL routine BREMPLOT\_THICK.PRO reads and plots the output from BREMTHICK. Photon flux is plotted as a function of photon energy in keV.

Following J. C. Brown, Solar Physics 18, 489 (1971), the photon flux at energy  $E_{ph}$  is computed from the equation

$$Flux = \frac{nNA}{4\pi(AU)^2} \frac{1}{(mc^2)^2} \int_{E_{ph}}^{E_{elHigh}} f(\gamma_o) v_o \left[ \int_{E_{ph}}^{E_o} \frac{\sigma(E_{ph}, E) v}{dE/dt} dE \right] dE_o$$

Here  $n$  and  $N$  are the number densities ( $\text{cm}^{-3}$ ) of the thermal plasma and energetic electrons, respectively.  $A$  is the area of the emitting region and  $AU$  is one astronomical unit (in cm), taken to be the distance from the source to the detector. The mass of the electron and the speed of light are  $m$  and  $c$ , respectively, and  $mc^2$  is the rest mass energy of the electron in keV.

The integrals are over electron energy,  $E$  or  $E_o$  (in keV).  $E_{elHigh}$  is the maximum energy of the radiating electrons. The electron distribution function is  $Nf(\gamma)$ , where  $\gamma$  is the relativistic gamma factor, related to the electron (kinetic) energy through  $E_{el} = (\gamma - 1)mc^2$ , and  $f(\gamma)$  is normalized so that

$$\int_{\gamma_{Low}}^{\gamma_{High}} f(\gamma) d\gamma = 1$$

The remaining terms in the integrand are  $v$ , the electron speed,  $\sigma(E_{ph}, E_{el})$ , the bremsstrahlung cross section, and  $dE/dt$ , the single electron energy loss rate. The cross section is from equation (4) of E. Haug, Astron. Astrophys. 326, 417 (1997). This cross section closely follows the relativistic cross section of Bethe and Heitler, equation 3BN in H.W. Koch and J. W. Motz, Rev. Mod. Phys. 31, 920 (1959), but requires fewer computations. The multiplicative Elwert correction to the Born approximation (G. Elwert, Ann. Physik 34, 178, 1939) is included in the cross section.

The energy loss rate,  $dE/dt$ , is assumed to be dominated by collisional losses. Following J. Leach & V. Petrosian, The Astrophysical Journal 251, 781 (1981), it is taken to be

$$\frac{dE}{dt} = 4\pi r_0^2 (mc^2) n c \ln \Lambda / \beta .$$

Here  $r_0$  is the classical radius of the electron,  $\ln \Lambda$  is the Coulomb logarithm, and  $\beta$  is  $v/c$ . Since collisional losses are proportional to the target number density, the plasma density,  $n$ , cancels out of the photon flux calculation.

The integrations are performed by Functions DMLINO and DMLINI using Gaussian quadrature. These function and Subroutines GAULEGO and GAULEGI in the file dmlinbr2.for are adapted from Press et al., Numerical Recipes in Fortran 77, Second Edition (Cambridge University Press, 1992). The input variable RERR determines the fractional error in the integration steps.

A double-power-law electron distribution function is provided by Subroutine DISTRN in the file fdblpbr.for. To facilitate the numerical integration when the photon energy, EPH, is less than the double-power-law break energy, EEBRK, BREMTHICK performs the outer integration in two parts, above and below EEBRK (or, equivalently, GAMBRK) and sums the parts. Similarly, Function FTHICK carries out the inner integration in two parts when appropriate.

In thick-target bremsstrahlung computations, the electron flux distribution (electrons  $\text{cm}^{-2} \text{s}^{-1} \text{keV}^{-1}$ ), rather than the electron density distribution (electrons  $\text{cm}^{-3} \text{keV}^{-1}$ ) is frequently specified. The code can be easily modified so that DISTRN provides the normalized flux distribution. In Function FTHICK( EO ), remove the variable BETA from the expression for FTHICK and replace it with the constant term  $1 / \text{CLIGHT}$ . Give the input parameter DENSTY the value of the total nonthermal electron flux density (electrons  $\text{cm}^{-2} \text{s}^{-1}$ ) instead of the number density.

This program, as provided, runs in approximately 3 seconds on a 500 MHz Pentium III Personal Computer with 128 MB of memory.

## **Contact**

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## Input Parameters

All input parameters are specified in Program BREMTHICK.

EELow	Low energy cutoff in the electron distribution function (in keV)
EEBRK	Break energy in the electron distribution function (in keV)
EEHIGH	High energy cutoff in the electron distribution function (in keV)
P	Power-law index of the electron distribution function below EEBRK
Q	Power-law index of the electron distribution function above EEBRK
Z	Mean atomic number of the target plasma
DENSTY	Number density of nonthermal electrons ( $\text{cm}^{-3}$ )
AREA	Area of the radiating source region ( $\text{cm}^2$ )
RERR	Desired relative error for evaluation of the integral. For example, RERR = 0.01 indicates that the estimate of the integral is to be correct to one digit, whereas RERR = 0.001 calls for two digits to be correct.
EPH	Photon energies (in keV) at which to compute the photon flux, specified in a DO loop.

## Output Parameters

EPH and FLUX are written to the file output.txt. EPH, IERGQL, IERGQH, and IERGQI are written to the file error.txt. All five of the output parameters are written to the screen as the program runs.

EPH	Photon energy in keV
FLUX	Photon flux at photon energy EPH in photons $\text{s}^{-1} \text{keV}^{-1} \text{cm}^{-2}$ . The detector is assumed to be 1 AU from the source.
IERGQL	Error flag from the outer Gaussian quadrature integration routine for integration over electron energies below EEBRK. The numerical integration converged if IERGQL = 0. It did not converge if IERGQL = 120.
IERGQH	Error flag from the outer Gaussian quadrature integration routine for integration over electron energies above EEBRK. The numerical integration converged if IERGQH = 0. It did not converge if IERGQH = 120.
IERGQI	Error flag from the inner Gaussian quadrature integrations. The numerical integrations converged if IERGQI = 0. Otherwise, the value of IERGQI is the number of integrations that did not converge.

## Subprograms

Program BREMTHICK	Specifies all input parameters, calls DMLINO to compute the photon flux for all photon energies, and writes the results to the screen and files.
Subroutine ELOSS	Computes the energy dependent terms of the energy loss rate, returned as DEDT. The numerical coefficient, DECOEFF, is computed in Program BREMTHICK.
Function FINNER	Computes the integrand for the inner numerical integration. Calls BREMCROSS and ELOSS.
Function FTHICK	Computes the integrand for the outer numerical integration. Calls DMLINI and FINNER to compute the inner integral. Calls DISTRN. The normalization coefficient, FCOEFF, is computed in Program BREMTHICK.
Subroutine BREMCROSS	Computes the bremsstrahlung cross section as a function of Z, incident electron energy (EEL), and radiated photon energy (EPH).
Subroutine DISTRN	Computes the normalized electron distribution function, $f(\gamma)$ . A double-power-law distribution function is provided.
Function DMLIN[O or I]	Performs the Gaussian quadrature numerical integrations. Repeatedly doubles the number of points evaluated until convergence, specified by RERR, is obtained or the maximum number of points, specified by the parameter MAXFCN, is reached. For DMLINO, IER = 120 is returned if the integration fails to converge. For DMLINI, IER = 1 is returned if the integration fails to converge. MAXFCN should be less than or equal to 2**NLIM, or 4096 with NLIM = 12.
Subroutine GAULEG[O or I]	Computes the Gaussian quadrature abscissas and weights for Function DMLIN[O or I]. Adapted from Press et al., Numerical Recipes in Fortran 77, Second Edition (Cambridge University Press, 1992).