

Recent Changes to the RHESSI Back Projection and Clean Algorithms and New Control Parameters

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Introduction

This document outlines the rationale for the new flat-fielding algorithm used in RHESSI back projection. It re-documents the current algorithm, outlines rate-based back projection, describes a compensation scheme for time-variable background, the rationale for the form and parameterization of this compensation, and summarizes the relevant control parameters. It applies to the Clean algorithm as well because the dirty map used in Clean is the data product of the Back Projection algorithm. At the end, we describe the following new control parameters for imaging introduced in Sept. 2004:

IMAGING_POWER_LAW
USE_RATE
USE_LOCAL_AVERAGE
LOCAL_AVERAGE_FREQUENCY
AUTO_FREQUENCY
USE_CULL
CULL_FRAC.

Current Back Projection Algorithm

For a given subcollimator, the flat-fielded back projection algorithm that has been in place since the start of the mission has been of the form:

$$S_n = \frac{\sum_i \{C_i (D_{in} - \langle D_n \rangle)\}}{AT \langle G \rangle \{\langle D_n^2 \rangle - \langle D_n \rangle^2\}} \quad (1)$$

where S_n is the estimated flux at the center of the n^{th} pixel; C_i represents the observed counts in the i^{th} time bin; A is the effective area of the detector (incorporating detector efficiency, attenuator and other absorbers, but not grid transmission); and T is the total livetime in seconds. The grid transmission probability for a source in pixel n for the i^{th} time bin is given by $G_i * D_{in}$, where G_i is the modulation-averaged grid transmission and D_{in} is of the form, $1 + \text{modamp} * \text{COS}(\text{geometric terms})$. $\langle G \rangle$ and $\langle D_n \rangle$ are the time-bin-averages (viz rotation-averaged) of grid transmission and modulation terms, respectively. Note that the modulation parameters typically depend on the distance from map center.

The key requirements for a good back projection algorithm are three-fold:

1. The algorithm should be linear.
2. For a single source in pixel, n , the expectation value of S_n should not depend on n . (viz, a flat gain)
3. The estimated value of S_n should not be dependent on any time-independent background.

Under most circumstances the algorithm has satisfied these conditions. However, two conditions have been identified in which this is not the case.

Rate-Based Back Projection

The first condition is when livetime corrections are both important and correlated with the modulation parameters. This can occur when the fractional livetime is significantly less than 1. To remedy this, the algorithm (Eq. 1) is being modified to become "rate-based back projection" so that

$$S_n = \frac{\sum_i \{R_i (D_{in} - \langle D_n \rangle)\}}{A \langle G \rangle \{\langle D_n^2 \rangle - \langle D_n \rangle^2\}} \quad (2)$$

where $R_i = C_i / t_i$, and t_i = the livetime in the i^{th} time bin. In this formulation, the averages, $\langle G \rangle$, $\langle D_n \rangle$ and $\langle D_n^2 \rangle$ are livetime weighted.

To avoid statistical 'spikes' when t_i happens to be very small due to data gaps, time bins are identified in which the livetime is less than a fraction (typically $\frac{1}{2}$) of the time bin size. These time bins are then discarded. This benign step is known as 'culling' with the fractional threshold specified by CULL_FRAC.

Rate-based back projection should improve results when livetime is sufficiently large that it is correlated with the modulation parameters. One way to think of the improvement is that rate-based back projection should yield, on average, the image expected from a perfect detector without livetime effects. The only downside will be a small second-order increase in the level of statistical noise.

Compensation for time-variable background

Although the current algorithm completely suppresses the effects of background when it is independent of time, the flat-fielding does introduce artifacts (in the form of rings around the rotation axis) when time-variations in the background are correlated with changes in the modulation amplitude. This seemingly unlikely condition occurs under the following three known circumstances:

1. The first is during integrations that last over a significant fraction of an orbit. In this case the aspect control system tends to systematically shift the rotation axis toward sun center during the orbit, changing its distance from map center and so changing the modulation parameters. In the meantime, the background may vary as the magnetic latitude is changing during a similar orbital timescale.
2. The second circumstance can occur when (for reasons explained above) the rotation axis shifts closer (or further) from the map center while the level of earth albedo background changes due to changing flare intensity.
3. The third circumstance (and probably the most common) is due to the 'coning' of the imaging axis about the rotation axis. This introduces a periodic (~4s) systematic variation in the modulation parameters as the imaging axis oscillates closer to and further from the map center. Simultaneously, there can be a corresponding periodic variation in background level due to the rotational term in the earth albedo component of background.

To compensate for this, the rate-based algorithm (Eq. 2) is modified to the form:

$$S_n = \frac{\sum_i \{(R_i - R'_i) D_{in}\}}{A \langle G \rangle \{ \langle D_n^2 \rangle - \langle D_n \rangle^2 \}} \quad (3)$$

where R'_i is a time-averaged rate associated with the i^{th} time bin. It can be thought of as a 'smoothed' modulation profile. This time-averaging is over a timescale between 1/16 and 1/4 rotation period that is large compared to the modulation period, yet short compared to the rotational timescale of the background variation.

This revised form is equivalent to Eq. (2) for time-independent background or for modulated sources, but eliminates the effect of any correlation between time-dependent background and modulation parameters.

The downsides of this formulation are a slight increase in statistical errors and a slight increase in computation time. Richard Schwartz has made the rather important point that using $R_i - R'_i$ instead of R_i as the observational constraint may enable MEM, Pixons and Forward Fitting to automatically be independent of background, without the need for explicit modeling.

Parameterization for calculation of the smoothed modulation profile

The requirements for calculation of the smoothed modulation profile are two-fold:

1. It should reproduce effectively all of the rotationally modulated background so that it will be fully subtracted from the original modulation profile.
2. It should not significantly degrade the amplitudes or bias the phases of the true modulation.

Assuming a 4-second rotation period, the longest modulation period that can be fully observed is 1 second. This occurs when the source is one grid-angular period away from the rotation axis (neglecting coning). Thus, the objective is an algorithm that will fully suppress signals with periods of 4 seconds or more, while fully retaining signals with periods of 1 second or less. Three options were considered.

1. A multipole digital high pass filter, (implemented through a combination of DIGITAL_FILTER and CONVOLVE). This was rejected because of concern that frequency-dependent phase shifts might be introduced plus uncertainties in how it would handle data gaps.
2. A rectangular smoothing algorithm (SMOOTH) with a suitable smoothing time. This had the disadvantage that its response to high frequencies was not monotonic $\sim \sin\{x\}/x$.
3. A Gaussian smoothing algorithm with a suitable smoothing time. This was adopted for now since its response to high frequencies is monotonic.

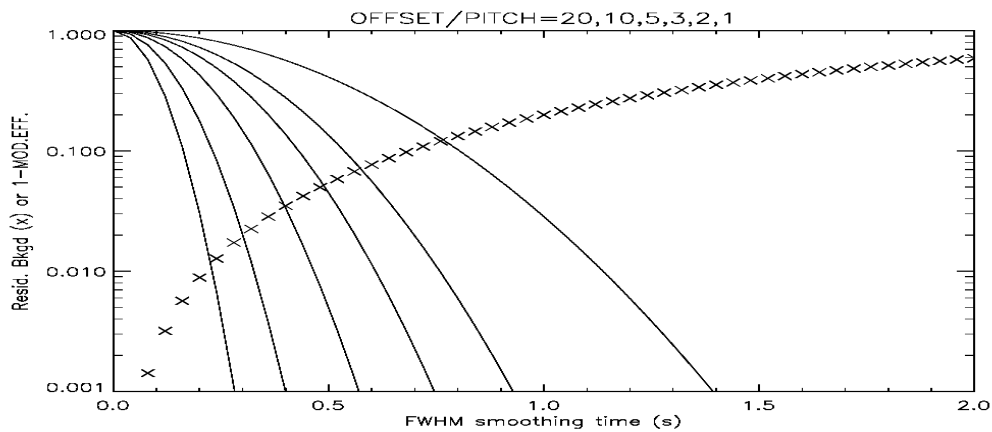
What is the optimum FWHM smoothing time for such a Gaussian? The plotted x's in the figure below show the residual rotationally-modulated background that would be left as a function of the FWHM smoothing time, H. It shows, for example that a smoothing time of 1.0 s would still leave 20% of the 4-s periodic term while a value of 0.25 s would leave only 1.4%, which would be quite satisfactory.

What is the effect of such smoothing on real modulation? The effect would be most severe on the slowest modulation cycle in each rotation. The period of this slowest cycle is given by:

$$1/(2\pi) * (\text{Rotation period}) * \text{ACOS}[1 - ((\text{angular pitch}) / \text{off-axis displacement})]$$

The solid curves in the figure below show the loss of modulation efficiency as a function of smoothing time for the six indicated values of the ratio, (off-axis displacement)/(angular pitch), with the left-most curve having the highest ratio. The figure shows, for example, that for grid 9 (6' pitch) and an off-axis displacement of 12', (i.e. a ratio of 2), a value of the FWHM smoothing time $H = 0.7$ s would reduce the modulation efficiency by about 2%.

For the coarsest grids, there is some compromise involved between suppressing rotationally-modulated background and retaining the full modulation amplitude. The figure shows that the true modulation is much more sensitive to H than is the level of background suppression. Therefore, the suggested compromise is that for each subcollimator the FWHM smoothing time is chosen to be equal to the period of the slowest modulation cycle. In this case, the maximum reduction in modulation amplitude is 2.8%. In addition, the shortest FWHM smoothing time can be 0.25 s since this eliminates all but 1.4% of the rotationally-modulated background. In the case where the map center is less than one angular pitch from the rotation axis, a complete modulation cycle will not be observed and the value of H should be sufficiently long (~ 4 s) so that it will have no effect. (It will still be effective on flare or orbital timescales, however.)



In summary then, the suggested algorithm for the calculation of the smoothed modulation profile is to convolve the original profile with a Gaussian of FWHM, H, where H is given by:

$$H = 4s * \text{MAX}(1/16, 1/(2\pi) * \text{ACOS}[1 - ((\text{angular pitch}) / \text{off-axis displacement})])$$

provided that the ratio (angular pitch / off-axis displacement) \leq 1.0
or H = 4s otherwise.

New Control Parameters

IMAGING_POWER_LAW – power-law index passed to *hessi_grm* used to compute modulation and transmission of x-rays through the grid based on the material properties weighted over the photon distribution. The default value is 4.0.

USE_RATE – this turns on the rate-enabled back projection described above. The default value is 0. This is particularly critical to set when the deadtimes become greater than 1/2. In such cases the deadtimes are strongly correlated with the modulated count rates and a significant amount of the imaging information may be encoded in the livetime variations (as opposed to the counts themselves). Because of this, for extreme cases, dirty maps made with *USE_RATE* set to 0 (the default) may show a strong negative (instead of positive) source at the true source location! Setting *USE_RATE* to 1 corrects that and other less obvious effects of high rates.

The next 3 control parameters work together.

USE_LOCAL_AVERAGE – this turns on the strategy described above that compensates for the time-variable background by averaging on a time scale shorter than the observation but long with respect to a modulation cycle. Default is 0.

LOCAL_AVERAGE_FREQUENCY- the values of the time width used in the Gaussian convolution in units of frequency with respect to the rotation period. It's a vector with dimensions of [9, 3] for each grid and harmonic. So a value of 16 for a rotation period of 4.0 seconds corresponds to a smoothing time of 0.25 seconds. The real default values will be set by the algorithm described above with *AUTO_FREQUENCY* set.

AUTO_FREQUENCY – uses the algorithm above to choose the values of *Local_Average_Frequency*. Default is 1. Depends on the grid pitch and distance of the source from the spin axis as these parameters determine the modulation period. There is a new info parameter *OFFAXIS_DISP* that is the average value in arcseconds of the distance from the source position (*XYOFFSET*) to the telescope axis.

The next 2 control parameters work together:

USE_CULL – turns on culling. Default is 1 for imaging, 0 for spectroscopy.

CULL_FRAC - culling fraction. Default is 0.5. With *USE_CULL* set, any time bin with a datagap deadtime larger than this fraction will have its count(s) value(s) set to 0 and its livetime set to 0.