

Principles of Forward Fitting

The intent of the Forward-Fitting process is to provide a parametric model which is a best fit to the observed count rate profiles. Ideally, this can be done without making maps of any kind. The idea is to find a model with the fewest parameters for a given χ^2 or C-statistic giving the "goodness-of-fit" of the computed model modulation profile to the observed profile. The model can be a superposition of circular Gaussians or a superposition of more complicated basis functions.

We restrict ourselves initially to models with two circular Gaussians. Such a model has 8 free parameters: $(x_1, y_1), (x_2, y_2)$ for the centroids of the sources, F_1, F_2 , for their fluxes, and w_1, w_2 for their widths. It is easy to show that the signal from a single sub-collimator cannot distinguish the difference between a point source and a circular Gaussian, so with little loss of generality, we can look at the case of two point sources.

Our strategy proceeds as follows:

1. Find the centroid of the source. This can be done in various ways:

- a) Find the best correlation between the single-revolution signal $f(\text{roll_angle})$ from the coarsest collimator and a function of the form:

$$f = \cos(A * \cos(\text{roll_angle} - B) - C)$$

This is very fast and the values of A and B lead to the azimuth and phase of the flare source.

OR:

- b) Make a coarse back-projection map using the two coarsest subcollimators, and find the peak of the map.

2. Make calibrated event lists for all collimators using the flare centroid for map center.

3. Using the profiles of the phase-map-ctr for each collimator, regularize the count rate and make corrections for gridtran, livetime and modamp variations. (Regularization is required so that the FFT may be used in the interpretation of $A_S^{(i)}(\nu)$.)

(*hsi_rebin.pro*, *hsi_regularize.pro*)

4. Compute the power spectrum $A_S^{(i)}(\nu)$, $i = 1 - 9$ of each of the 9 regularized profiles.

5. Determine the location of significant peaks in the appropriate ranges of each $A_S^{(i)}(\nu)$. Look for double peaks, starting with the coarsest collimators and working towards the finest. At a given pitch, source separations greater than that pitch will produce a double peak. This will usually be seen in signals from 3 successive collimators with pitches spanning the separation size. The separation of the sources can then be directly determined from the power-spectral maxima (ν_1, ν_2) .

6. For collimators with $|\nu_1 - \nu_2|$ significantly > 0 there will be a slow amplitude modulation ("beating") of the signal. The beat signal is best understood as a long-wavelength waveform in the U-V plane, which modulates the visibilities. (See Figure) The beat waveform, like the parent visibility, has amplitude, phase and direction. The direction of this beat waveform is the same as the direction of the separation vector for the source pair. In general this direction is not found directly from a single power spectrum $A_S^{(i)}(\nu)$, because the signal has been regularized from the point of view of the flare centroid, which is generally a different direction than the separation vector,

Given the period $|\nu_1 - \nu_2|$ and the mean frequency $\bar{\nu} = (\nu_1 + \nu_2)/2$ one may find the separation vector by using the Hilbert transform. But first one must filter the spectrum.

- a) Fourier filter the signal by using only spatial frequencies whose spectral power exceeds 3σ .
- b) Compute the complex signal ($i = \sqrt{-1}$)

$$f(\hat{k}) = f(k) + i \text{HILBERT}(f(k)),$$

where HILBERT is a function in the IDL 5.x library.

- c) Then eliminate the high frequencies by a phase shift:

$$g(k) = f(\hat{k}) \times e^{i\bar{\nu}k}$$

where the shift is given by the peak frequency of the power spectrum.

For two circular Gaussian sources with a radial separation vector, $g(k)$ will be a low-frequency sinusoid.

If the beat amplitude is large (i.e. nearly equal sources), and the separation happens to be radial, the amplitude and phase can be found immediately from $g(k)$. Since in general the separation is nonradial, which causes the the beat signal to have a period longer than the true period. It must be found from two orthogonal regularizations:

- a) Find the beat frequency ν_1 from regularization #1.
- b) Perform the orthogonal regularization.
- c) Find the beat period ν_2 from regularization #2.
- d) The true beat frequency is the quadrature sum, $\sqrt{\nu_1^2 + \nu_2^2}$.
- e) The separation, direction and centroid give x_1, x_2

7. The total flux on the coarsest scales provides the sum $F_1 + F_2$.

8. The collimator-to-collimator ratios of the spectral power in each source give the ratios of a Gaussian function of the widths w_i and fluxes F_i , from which the final fluxes and widths may be determined.

Once the parameters have been found, the model count rate can be compared with the observed one, and the χ^2 or C-statistic can be computed. If these are sufficiently small, we are done. If not, small adjustments of the parameters can be made in a search for a better fit. If this does not lead to a satisfactory statistic, it will be necessary to search for a third source or more complicated sources.