Notes on Continuum Radiative Transfer and Emission Mechanisms

T. S. BASTIAN
National Radio Astronomy Observatory
Charlottesville, VA

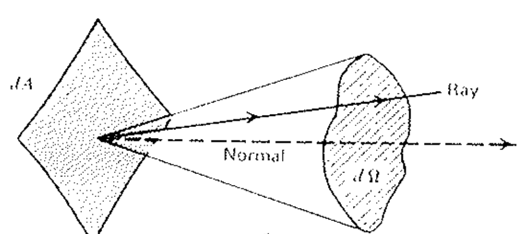
December 4, 2005

These notes briefly summarize some ideas related to the production and transfer of continuum radiation at radio wavelengths. By continuum radiation I mean broadband radiation ($\Delta\nu/\nu > 1$) which varies continuously and gradually with frequency ν. By contrast, emission or absorption in spectral lines is such that the width of line is much less than the radio frequency at which it is emitted (or absorbed): $\Delta\nu_L \ll \nu$. I begin with some definitions and concepts useful to radiative transfer, set up the transfer equation, and give the formal solution. I'll then discuss two common emission mechanisms which produce continuum radiation at radio wavelengths – bremsstrahlung and gyromagnetic radiation. I'll conclude by mentioning some effects of the background medium on radiation and its transfer. For those interested in pursuing the subject in more detail, I suggest reading the excellent introductory book *Radiative Processes in Astrophysics* by Rybicki & Lightman or *Radiation Processes in Plasmas* by Stix.

1 Some Useful Definitions and Concepts

1.1 Specific Intensity, Brightness, Flux Density

For the moment, think of radiation as propagating along rays. The radiation flux is a measure of radiative energy carried by all rays passing through a given area. Consider a small area dA perpendicular (normal) to a given ray (Fig. 1). Now consider all rays passing through dA whose direction is within a solid angle $d\Omega$ of the given ray.



The energy crossing dA in a time dt and a frequency range $d\nu$ is defined by

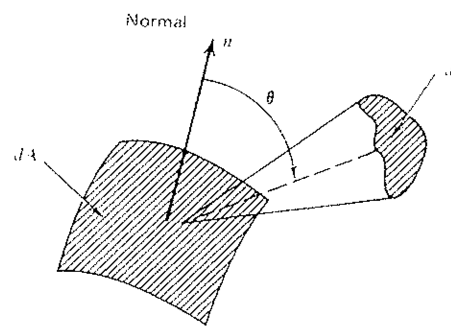
$$dE = I_\nu \; dA \; dt \; d\Omega \; d\nu$$

where I_ν is the *specific intensity* or *brightness*. The units of I_ν are energy/(area · time · solid-angle · frequency), or ergs cm^{-2} s^{-1} ster^{-1} Hz^{-1}.

Return to the small area dA. The flux of radiation through dA per second per Hz from a solid angle $d\Omega$ is the specific intensity times the solid angle $d\Omega$, but reduced by $\cos\theta$ where θ is the angle between the normal of dA and the ray directed along $d\Omega$ (Fig. 2). So we have $dF_\nu = I_\nu \cos\theta d\Omega$. Integrating over all directions (i.e., all solid angle) we have

$$F_\nu = \int I_\nu \cos\theta \; d\Omega.$$

Note that if the radiation is isotropic (the same in all directions), the net flux of radiation though dA is zero.



Now consider a distant source observed by a radio telescope. If the source is resolved, we have a measure of the radiation flux per resolution element. The resolution element, determined by the properties of the instrument, subtends a solid angle $\Omega_\circ$. We often call $\Omega_\circ$ the "beam" in radio astronomy. Then, because $F_\nu = I_\nu \Omega_\circ$, $I_\nu = F_\nu / \Omega_\circ$. We call this the *flux density*. It represents an imperfect measure of the specific intensity of the source – imperfect because we are limited by the finite angular resolution $\Omega_\circ$ of our instrument. At radio wavelengths, we measure flux in units of Janskys, in honor of Karl Jansky, the Bell Labs engineer that first discovered radio signals of cosmic origin: 1 Jy = 10^{-26} ergs cm^{-2} s^{-1} Hz^{-1} = 10^{-23} W m^{-2} Hz^{-1}. Hence, the unit of flux density is commonly Jy/beam.

1.2 Emission and Absorption Coefficients

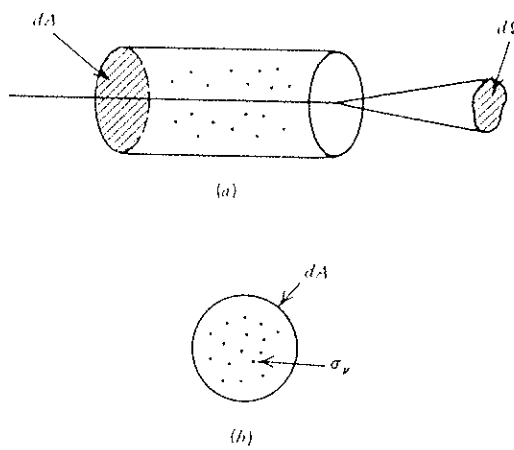
It is important to point out that rays traveling in free space have a constant specific intensity. In other words, $dI_\nu/ds = 0$. Therefore only emission and absorption of the ray can change the specific intensity.

The *emission coefficient* is defined as the energy emitted per unit time per unit solid angle per unit volume per unit frequency: $dE = j_\nu \; dV \; d\Omega \; dt \; d\nu$. It therefore has units of ergs cm^{-3} s^{-1} ster^{-1} Hz^{-1}. It is easy to see that for rays traveling a distance ds, a beam of cross section dA travels through a volume $dV = dA \; ds$ and the intensity added to the beam is therefore $dI_\nu = j_\nu \; ds$.

The beam looses energy by absorption as it travels a distance ds. The *absorption coefficient* is defined by $dI_\nu = -\alpha_\nu \; I_\nu \; ds$ where the convention is $\alpha_\nu > 0$ for energy removed from the beam. To see this phenomenologically, consider a random distribution of absorbing particles with a density n in a some volume. Suppose the effective absorbing area[1] (cross section) of each particle is σ_ν cm^2. For an area dA and a distance ds, the volume is $dA \; ds$, the number of absorbing particles is $n \; dA \; dS$, and the total absorbing area is $dA_{eff} = n\sigma_\nu \; dA \; ds$. Then the energy removed from the beam by the absorbers is

$$I_\nu \; dA_{eff} \; d\Omega \; dt \; d\nu = I_\nu \; (n\sigma_\nu dAds) \; d\Omega \; dt \; d\nu.$$

So, $dI_\nu = -n\sigma_\nu \; I_\nu \; ds$ and we identify $\alpha_\nu = n\sigma_\nu$ in the present case.



For the purposes of this simple discussion, I have ignored scattering processes, which can scatter radiation into or out of the beam. I have also ignored stimulated

[1] This treatment is only valid if the linear size of the absorbing particles is much less than the distance between them, a condition almost always met in astrophysical applications.

emission, which can be considered a "negative absorption" because it, like normal absorption, depends on the specific intensity.

2 Radiative Transfer

2.1 The Radiative Transfer Equation

We now have all the pieces to write down an equation which describes the change in specific intensity along a ray:

$$dI_\nu = -\alpha_\nu I_\nu ds + j_\nu ds \longrightarrow \frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu.$$

This is the *radiative transfer equation*. It describes the macroscopic behavior of radiation in an emitting and absorbing medium, hiding all of the microscopic physics in α_ν and j_ν (which I'll get to below). It is useful to recast the equation in a more intuitive form using the *optical depth* and the *source function*. The optical depth τ_ν is defined by $d\tau_\nu = \alpha_\nu ds$, or

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')ds'.$$

Note that τ_ν is dimensionless. The *source function* is defined as the ratio of the emission and absorption coefficients: $S_\nu = j_\nu/\alpha_\nu$. We can then rewrite the transfer equation as

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu.$$

The formal solution to the transfer equation is

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu) d\tau'_\nu.$$

Consider the simple case of a constant source function S_ν. Then the solution becomes

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

It's easy to see that as τ_ν becomes large, I_ν aproaches S_ν. More generally, if $I_\nu > S_\nu$, $dI_\nu/d\tau_\nu < 0$ and I_ν decreases along the the ray; if $I_\nu < S_\nu$, $dI_\nu/d\tau_\nu > 0$ and I_ν tends to increase along the the ray. Hence, the specific intesity always approaches the source function given sufficiently large τ_ν. We refer to media where $\tau_\nu \gg 1$ as being *optically thick* and those where $\tau_\nu \ll 1$ as being *optically thin*.

2.2 Thermal Radiation and Brightness Temperature

Before continuing, it's worth a brief aside to discuss thermal radiation. Thermal radiation, sometimes called *blackbody radiation* when it is optically thick, is in thermal equilibrium. Consider an enclosure which is held at some temperature T. No radiation is let in or out of the enclosure. After some time, the interior of the enclosure is filled with radiation in thermal equilibrium with the walls. Suppose we made a small hole in the enclosure and measured the radiation without disturbing the equilibrium. It so happens that the specific intensity is fully specified under these conditions by a universal function of T and ν; i.e., $I_\nu = B_\nu(T)$. The function $B_\nu(T)$ is called the Planck function. A derivation of the Planck function, based on thermodynamic and quantum mechanical arguments, can be found in Rybicki & Lightman. It is given by

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

where h is Planck's constant and k is Boltzmann's constant. Thermodynamic arguments also establish that $S_\nu = B_\nu$ so that $j_\nu = \alpha_\nu B_\nu(T)$ (Kirchhoff's Law). At radio wavelengths the frequency of the radiation is such that $h\nu \ll kT$. We can therefore expand the exponential in the Planck function: $e^{h\nu/kT} - 1 \approx h\nu/kT$. Therefore, to a high degree of precision we have the simple expression referred to as the *Rayleigh-Jeans Law*:

$$B_\nu(T) = I_\nu(T) \approx \frac{2\nu^2}{c^2} kT.$$

Given the simplicity of the expression for the specific intensity in the Rayleigh-Jeans regime, it is useful to characterize the brightness at a particular frequency by the temperature of the blackbody having the same brightness at that frequency. We refer to this temperature as the *brightness temperature* T_B; it is defined through the expression

$$I_\nu = B_\nu(T_B) = \frac{2\nu^2}{c^2} kT_B.$$

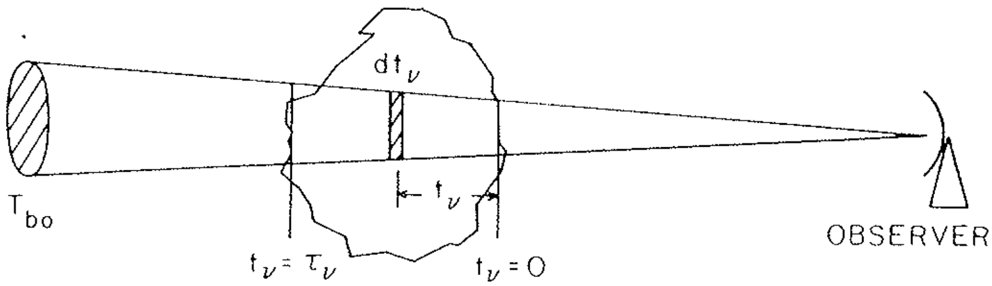
In addition to having the advantage of being related to the physical properties of the source of radiation, it also has the advantage of simple units (Kelvin, as opposed to ergs cm^{-2} s-1 ster^{-1} Hz^{-1}). Given the relationship between I_ν and T_B, S_ν and T, we can rewrite the transfer equation:

$$\frac{dT_B}{d\tau_\nu} = -T_B + T$$

which, when T is constant, has the solution

$$T_B = T(1 - e^{-\tau_\nu})$$

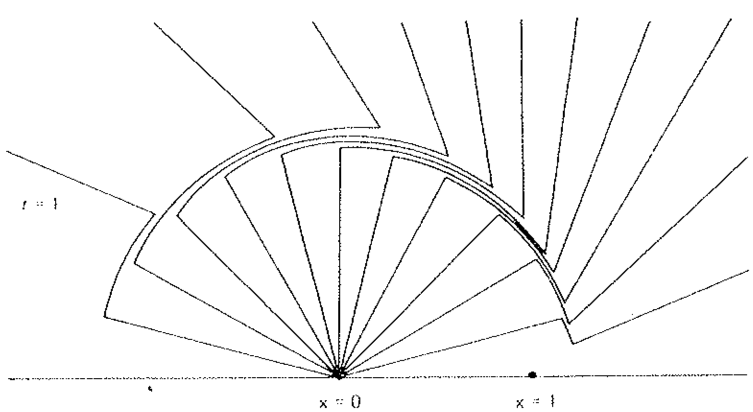
in the absence of background emission. Note that when the source is optically thick $T_B = T$; when the source is optically thin the exponential can be expanded and we have $T_B \approx \tau_\nu T$.



3 Radio Continuum Emission Mechanisms

3.1 Preliminaries

Continuum radiation is produced by accelerating charge. To see this in detail is far beyond the scope of this lecture, but one can get an intuitive inkling of why this might be by refering to the figure. It shows the electric field of a particle which has moved uniformly in the $+x$ direction until, at time $t = 0$ and location $x = 0$, it is stopped abruptly. Because of the finite speed of light, points far from the particle have not yet received the news that it has stopped and the electric field is that of the moving particle. But inside the radius $r = ct$, the field is that of the motionless particle. Hence, a transverse disturbance has formed which moves out from the particle at light speed.



3.3 Gyromagnetic Radiation

Collisions between electrons and ions is not the only way to produce continuum radio radiation. If a magnetic field is present in a plasma the electrons experience the Lorentz force:

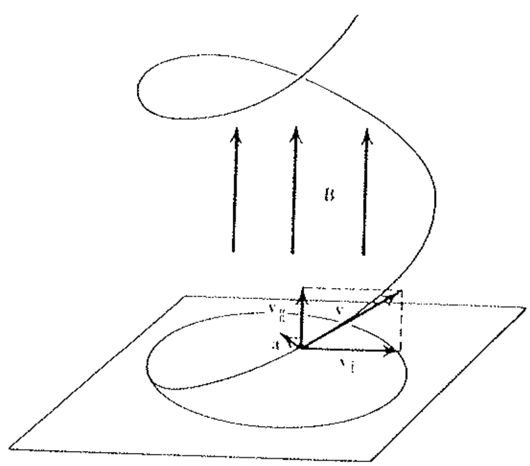
$$F = \frac{e}{c}\mathbf{v} \times \mathbf{B} \quad \text{or} \quad m_e\ddot{\mathbf{r}} = \frac{e}{c}\dot{\mathbf{r}} \times \mathrm{B}$$

Here e is the charge of an electron, c is the speed of light, $\mathbf{v}$ is the electron velocity and $\mathbf{B}$ is the magnetic field. Suppose we have a volume permeated by a uniform magnetic field in the $\hat{\mathbf{z}}$ direction: $\mathbf{B} = (0, 0, B_z)$. Consider an electron moving at a constant speed perpendicular to the magnetic field. If its initial velocity vector is $\mathbf{v} = (0, v_\perp, v_z)$ and its initial position is $\mathbf{r} = (x_o, y_o, z_o)$ we can write an equation of motion for each coordinate and solve it:

$$x(t) = x_o + \frac{v_\perp}{\Omega_e}(1 - \cos\Omega_e t)$$

$$y(t) = y_o + \frac{v_\perp}{\Omega_e}\sin\Omega_e t$$

$$z(t) = z_o + v_z t$$



The electron moves in a helical path; if $v_z = 0$ it moves in a circular path a frequency Ω_e. We refer to $\Omega_e = eB/m_e c$ as the *electron gyrofrequency*. Similarly, a positively charged particle also has a circular motion in a magnetic field, but in the opposite sense of an electron. Furthermore, the gyrofrequency of an ion is given by

A detailed treatment of the nonrelativistic ($v \ll c$) case of a charge q with acceleration $\dot{\mathbf{v}}$ yields the *Larmor formula* for the power emitted by a single accelerating particle of charge e into solid angle $d\Omega$:

$$\frac{dP}{d\Omega} = \frac{q^2 \dot{\mathbf{v}}^2}{2c^3} \sin^2 \Theta$$

Here Θ is the angle relative to the vector along which the particle is accelerated. Note that the power radiated is proportional to (charge$\times$acceleration)^2, that the *radiation pattern* is dipolar (the $\sin^2 \Theta$ pattern), and that the emission is peaked in the direction perpendicular to the acceleration vector.

The relativistic case (i.e., charged particles moving at a significant fraction of light speed) requires a thorough grounding in the theory of special relativity. The relativistic counterpart to the above equation for the angular distribution of the power emitted is

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} \frac{(\gamma^2 a_\parallel^2 + a_\perp^2)}{(1-\beta\mu)^4} \sin^2 \Theta$$

where $a_\perp$ and $a_\parallel$ is the acceleration perpendicular and parallel to the velocity vector of the charged particle, respectively, and $\gamma = 1/\sqrt{1-(v/c)^2}$ (the *Lorentz factor*). The chief effect of relativistic particle speeds is beaming. The figure shows the special cases of beaming for acceleration parallel to or perpendicular to the instantaneous velocity of the charged particle.

For an ensemble of charged particles, the power emitted has the same form as the single particle case under the *dipole approximation*. However, rather than $\dot{\mathbf{v}}^2$, we have $\ddot{\mathbf{d}}^2$ where $\mathbf{d} = \sum_i e_i \mathbf{r}_i$ is the dipole moment of the ensemble.

3.2 Bremsstrahlung or Free-Free Radiation

Bremsstrahlung is a German word meaning "braking radiation". The term free-free radiation is also often used, referring to the fact that unbound charged particles are involved. Specifically, bremsstrahlung radiation is due to the interaction between ions and electrons. When an electron passes near an ion, both particles experience a deflection due to the Coulomb force. The deflection of the particle trajectories is an acceleration and the particles therefore radiate. The deflection is inversely proportional to the mass of the particles and is therefore much larger for electrons than ions. Hence electrons are the main emitters in bremstrahlung and the contribution of ions may be neglected – that is, one can simply treat the problem in terms of the deflection of the electron by the electric field of the ion.

The way the calculation proceeds is as follows: 1) the radiation from a single electron moving past an ion with an impact parameter b is calculated; 2) the contributions from all electrons passing all ions over all impact parameters are then summed. This means summing over all electron velocities in a distribution and all (relevant) impact parameters. The range of impact parameter b over which one integrates is a little tricky. The minimum impact parameter b_{min} depends on some quantum mechanical considerations, and the maximum impact parameter b_{max} is where the effect of the ionic electric field becomes negligible. The most important case encountered in astrophysics is thermal plasma, where the velocity distribution of the electrons and ions is *Maxwellian*. In this case, the probablility of a particle having a speed v in the range dv is

$$dP \propto v^2 e^{-mv^2/2kT} dv,$$

and one calculates the *thermal bremsstrahlung* or *thermal free-free* emission by summing (integrating) over the contributions of all particles in the Maxwellian distribution function.

The (angle-integrated) emissivity resulting from electron-ions collisions in a thermal distibution of particles is:

$$\epsilon_\nu^{ff} = \frac{2^5 \pi e^6}{3 m_e c^3} \Big(\frac{2\pi}{3 k m_e} \Big)^{1/2} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} g_{ff}(\nu, T)$$

where $\epsilon_\nu^{ff} = 4\pi j_\nu^{ff}$. n_e is the electron number density, n_i is the ion number density, and Z is the ion charge. $g_{ff}(\nu, T)$ is called the *Gaunt factor*, a slowly varying function of T and ν. It incorporates the range of impact parameters important to the problem:

$$g_{ff}(\nu, T) = \frac{\sqrt{3}}{\pi} \ln \Big(\frac{b_{max}}{b_{min}} \Big)$$

For thermal free-free emission, we can use Kirchhoff's Law to derive the absorption coefficient. Doing so, one obtains

$$\alpha_\nu^{ff} = \frac{4e^6}{3mhc} \Big(\frac{2\pi}{3km} \Big)^{1/2} T^{-1/2} Z^2 n_e n_i \nu^{-3} (1 - e^{h\nu/kT}) g_{ff}(T, \nu)$$

Note that the exponentials in both expressions can be expanded in the Rayleigh-Jeans (radio) regime. For the absorption coefficient we get the much simpler expression:

$$\alpha_\nu^{ff} = 0.018 T^{-3/2} Z^2 n_e n_i \nu^{-2} g_{ff}(T, \nu)$$

It is also worth noting that the source function is simply the Plankian. Hence the frequency dependence of the source function is ν^2 in the radio regime.

$\Omega_i = ZeB/m_i c$. A proton is almost 2000 times more massive than an electron. It gyrofrequency is therefore almost 2000 times smaller than that of an electron in a given magnetic field.

Because a particle in a magnetic field moves in a circular trajectory (or a helical trajectory if it has a velocity component v_z) the electron is continuously accelerated. It therefore emits photons. The general expressions for the emission and absorption coefficients of gyromagnetic radiation are very complicated. Luckily, various approximations can be made when the electrons involved are non-relativistic or ultra-relativistic. We refer to radiation produced by electron gyration in a magnetic field in the former regime as gyroresonance or cyclotron radiation; in the high energy regime we refer to synchrotron radiation; and in the intermediate regime we refer to gyrosynchrotron radiation.

3.3.1 Gyroresonance or Cyclotron Radiation

When the electron moves in a magnetic field at non-relativistic speeds ($v \ll c$) it emits radiation at discrete frequencies which are low *harmonics* of the gyrofrequency. In other words, the cyclic frequency of the radiation emitted is at $s\nu_B$ where $s = 1, 2, 3, ...$ and $\nu_B = \Omega_e/2\pi$. The figure shows the radiation pattern of various low harmonics of the (cyclic) gyrofrequency ν_B.

Because gyroresonance involves low-energy electrons, it typically involves a thermal distribution of electrons. An example of thermal gyroresonance is found on the Sun. Strong thermal gyroresonance emission is often seen above sunspots where the magnetic field is strong. The approximate expression for the the thermal gyroresonance absorption coefficient is:

$$\alpha_\nu^{gr} = \frac{4\pi^{3/2}}{c}\frac{\nu_{pe}^2}{\nu}\frac{(s/2)^{2s}}{s!}\frac{(\sin\theta)^{2s-2}(1+\cos^2\theta)}{\cos\theta}\beta_\circ^{2s-3}\exp\Big[-\frac{(1-s\nu_B/\nu)^2}{\beta_\circ^2\cos^2\theta}\Big]$$

Here $\nu_{pe} = \sqrt{e^2 n_e/\pi m_e}$ is the electron plasma frequency (see below) and $\beta_\circ = 2kT/m_e c^2$. Since the electron distribution function is thermal, one can again relate the absorption coefficient to the emission coefficient through Kirchhoff's Law.

3.3.2 Gyrosynchrotron Radiation

When the emitting electrons are mildly relativistic, so that their kinetic energy is comparable to their rest mass (energies of order 100s of keV to a few MeV), and the Lorentz factor γ is of order a few, the electrons emit at higher harmonics of the electron gyrofrequency – somewhere in the range of $s \sim 10 - 100$. The emission is no longer characterized by discrete harmonics – the harmonics broaden and merge with increasing energy so that a continuum is formed (Fig.?).

Horrifying as α_ν^{gr} is above, it is much worse for case of intermediate electron energies. The emission and absorption coefficients cannot be written down in a convenient analytic form in this case. This is because they involve sums over many harmonics of the gyrofrequency. For this reason detailed calculations of gyrosynchrotron emission can be quite tedious and approximate forms are often used. Examples may be found in the review by Dulk (1985, Ann. Rev. Astron. Astrophys., 23, 169).

Gyrosynchrotron emission is encountered on astrophysical objects where the magnetic field is intermediate to strong: 10s to 100s of Gauss. Such magnetic field strengths are commonly encountered on the Sun and active stars. The electron distribution function may be thermal, in which case one refers to *thermal gyrosynchrotron* radiation; or the distribution function may be nonthermal, in which case one refers to *nonthermal gyrosynchrotron* radiation. The most common nonthermal electron distribution encountered in practice is the power-law, where the number $N(E)$ of electrons with energies between E and $E + dE$ is given by $N(E)dE = CE^{-\delta}dE$. δ is the spectral index of the electron energy distribution.

3.3.3 Synchrotron Radiation

When the emitting electrons are fully relativistic, $\gamma \gg 1$ and we refer to synchrotron radiation. In this case, electrons emit a broadband continuum of radiation at high harmonics (around $s = \gamma^2$ of the gyrofrequency. Because relativistic electrons are involved, the emission from a single electron is highly beamed with an angular width of order $\theta \approx 1/\gamma$.

In the case of relativistic particle distributions, nonthermal distributions are the rule. Mechanisms which heat plasmas in bulk to relativistic energies are not encountered in practice. Synchrotron emission from nonthermal (usually parameterized as power-law) distributions are ubiquitous in astrophysics. The beautiful luminous radio lobes found in radio galaxies are due to emission by ultrarelativistic electrons interacting with very weak magnetic fields (of order 10 microgauss). The absorption coefficient for synchrotron radiation from a power-law distribution of electrons is unpleasant but manageable:

$$\alpha_\nu^{syn} = \frac{\sqrt{3}e^3}{8\pi m_e}\Big(\frac{3q}{2\pi m_e^3 c^5}\Big)^{\delta/2} C(B\sin\alpha)^{(\delta+2)/2}\Gamma\Big(\frac{3\delta+2}{12}\Big)\Gamma\Big(\frac{3\delta+22}{12}\Big)\nu^{-(\delta+4)/2}$$

where α is the electron pitch angle, and the Γs refer to the Γ function. Note the fairly simple dependence of the the absorption coefficient on frequency. The emission coefficient is proportional to $\nu^{(\delta-1)/2}$ so the source function depends on frequency as $S_\nu = j_\nu/\alpha_\nu \propto \nu^{5/2}$.

3.4 Other Mechanisms

Thermal bremstrahlung and synchrotron radiation are the continuum emission mechanisms most commonly encountered at radio wavelengths in astrophysics. There are many other mechanisms, though, which may be important for certain kinds of sources. I'll mention two: plasma radiation and inverse Compton radiation.

I have already discussed gyromagnetic emission. Electrons oscillate in a magnetic field and therefore emit directly into harmonics of the electron gyrofrequency. There are other types of oscillations in astrophysical plasmas. One is plasma oscillations, or Langmuir waves, caused by the electric field between ions and electrons. The frequency of oscillation is called the *electron plasma frequency* which was already encountered above in the gyroresonance absorption coefficient: $\nu_{pe} = \sqrt{n_e e^2/\pi m_e}$, where n_e is the electron number density). Electrostatic oscillations can be excited in a plasma by a variety of mechanisms, including electron beams, shocks, and anisotropies in the electron distribution function. Langmuir waves are longitudinal waves (like sound waves) as opposed to electromagnetic waves which are transverse waves. Unlike electromagnetic waves, electrostatic waves cannot exist in a vacuum – they can only exist in a plasma. However, various mechanisms can convert electrostatic waves into electromagnetic waves. For example, a Langmuir wave scattering off of an ion can be converted into an electromagnetic wave with a frequency ω_{pe}; or two Langmuir waves colliding head on can produce a photon with a frequency $2\omega_{pe}$. Since $\omega_{pe}/2\pi \approx 9000\sqrt{n_e}$, largish densities (say, $10^8 - 10^10$ electrons per cm^3) are required to produce plasma radiation at wavelengths accessible to an instrument like the VLA (which offers frequency coverage from 300 MHz to 50 GHz). These sorts of densities are only encountered in stellar atmospheres or exotic environments like accretion disks around compact objects such as white dwarfs, neutron stars, and black holes.

Another mechanism of interest – not strictly relevant to the production of radio photons – involves photons scattering from free electrons. While the physical mechanism is the same, one often sees *Thomson scattering*, *Compton scattering*, and *inverse Compton scattering* discussed in the literature depending on the energy of the electrons or photons in question. Thomson scattering is the classical treatment of scattering by free electrons; Compton scattering involves low-energy electrons and high-energy photons; inverse Compton scattering involves low-energy photons and energetic electrons. Many astrophysical source involve energetic electrons (e.g., AGNs), in which case inverse Compton scattering is relevant. The importance of inverse Compton scattering is that it offers a mechanism whereby energetic electrons transfer energy to photons. In fact, the photon's energy is increased by a factor γ^2! For certain radio sources, inverse Compton scattering imposes a *limit* on their brightness. The compact cores of AGNs are thought to be due to self-absorbed (optically thick) synchrotron emission. Synchrotron emission is due to ultrarelativistic electrons. When the energy density of photons becomes sufficiently high inverse

Compton scattering becomes operative and degrades the energy of the very electrons producing the background photons!

4 Modifications Due to the Presence of a Medium

In practice, radiation does not propagate in a true vacuum. Radiation propagating from a distant source must traverse the interstellar medium, which contains contains gas and dust; the interplanetary medium, which is composed of a tenuous plasma moving away from the Sun at supersonic speeds (the solar wind); the Earth's ionosphere, caused by the bombardment of the upper atmosphere by energetic photons and particles from the Sun; and the Earth's atmosphere which is dense and neutral. All of these media affect radio waves in different ways and to varying degrees. In many cases the plasma or gas through which the radiation propagates is so tenuous that it has no discernible effect. But in other cases it does and it's effects may be parameterized in the *refractive index* of the medium, which we denote μ_ν. I will simply write down the modified transfer equation:

$$\frac{d}{d\tau_\nu}\left(\frac{I_\nu}{\mu_\nu^2}\right) = -\frac{I_\nu}{\mu_\nu^2} + S_\nu.$$

For an unmagnetized plasma, $\mu_\nu = \sqrt{1 - \nu_{pe}^2/\nu^2}$. Note that for $\nu < \nu_{pe}$ the refractive index is imaginary and electromagnetic waves cannot propagate!

In a magnetized medium, μ_ν has a more complicated form which depends on the polarization of the electromagnetic wave. When the frequency of the wave $\nu \gg \nu_{pe}, \nu_B$, the effect of the medium is largely limited to *Faraday rotation.* Faraday rotation of linearly polarized radiation is due to the fact that right- and left-hand circularly polarized modes which combine to form the linearly polarized component get out of phase as they propagate through a rarefied and weakly magnetized medium. As a result, the plane of the electric field vector rotates. Faraday is an important observable at radio wavelengths.