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The Imaging Capabilities of HESSI

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1 Introduction

The only practical method of obtaining ∼arc-second angular resolution in hard X-rays and gamma-rays within the cost, mass, and launch constraints of a small satellite is to use Fourier-transform imaging. (See Prince *et al.* 1988 for a review of imaging techniques.) One of the most powerful of the Fourier family techniques is rotational modulation synthesis, first proposed by Mertz (1967) and implemented by Schnopper *et al.* (1968). Rotational modulation was first used for solar flare X-ray imaging by the Japanese Hinotori telescope (Makashima *et al.* 1977, Ohki *et al.* 1982, Enome 1982), with angular resolution of $28''$ in the 20-40 keV energy range. A later Japanese telescope, HXT, on the Yohkoh satellite, used non-rotating Fourier synthesis with angular resolution of $\sim 8''$ in the 20-100 keV energy range (Kosugi *et al.* 1991). A prototype of a rotation modulation telescope–the High Energy Imaging Device (HEIDI)– was flown in 1993 on a balloon, without successful flare imaging, but with a definitive test of a novel solar aspect system (SAS), proving 0.5-arcsecond performance at balloon altitudes. (Crannell *et al.* 1993)

The High Energy Solar Spectroscopic Imager (HESSI) also uses rotational modulation synthesis imaging. With a launch in July, 2000 near the peak of flare activity, HESSI has been designed to image hard X-ray and gamma-ray flares from spatial resolution of $2.3''$ and spectral resolution of ~ 1 keV (Lin *et al.* 1993, 1994, 1998, Holman *et al.* 1997). We summarize briefly the HESSI instrumentation used to exploit Fourier synthesis imaging.

1.1 The HESSI Imager

HESSI uses nine bi-grid collimators, each consisting of a pair of widely separated grids in front of an X-ray/gamma-ray detector. Each grid consists of a planar array of equally-spaced, X-ray-opaque slats separated by transparent slits. The slits of each pair of grids are parallel to each other and their pitches (p) are identical, so the transmission through the grid pair depends on the direction of the incident X-rays. (See Figure 1.) For slits and slats of equal width, the transmission is modulated from zero to 50% and back to zero for a change in source angle to collimator axis (orthogonal to the slits) of p/L where L is the separation between grids (1550 mm). The angular resolution is then defined as p/(2L). For HESSI, the transmission of the source photons through the grids is modulated by the rotation of the spacecraft at about 15 RPM. The detector records the arrival time and energy of individual photons from anywhere on the Sun, allowing the modulated counting rate to be determined as a function of rotation angle.

Note that the detectors have no spatial resolution and hence have been optimized for high sensitivity and energy resolution. The nine segmented Ge detectors (GeDs), one behind each RMC, detect photons from $\sim$3 keV to 15 MeV. The GeDs are cooled to $\sim$ 75 K by a space-qualified long-life mechanical cryocooler. As the spacecraft rotates, the RMCs convert the spatial information from the source into temporal modulation of the photon counting rates of the GeDs. The instrument electronics amplify, shape, and digitize the GeD signals, provide power, format the data, and interface to the spacecraft electronics.

HESSI exploits one of the fundamental efficiencies of Fourier imaging: precision spacecraft pointing can be traded for exact knowledge of pointing. With a Fourier imager, it is not necessary to stabilize the spin axis to any better than a few arc minutes, as long as the system gives sufficiently precise and high-bandwidth pointing information. For HESSI, this information is provided by the solar aspect system (SAS) – a heritage of the HEIDI SAS – and the roll angle system (RAS). The SAS consists of 3 linear diode arrays on which the full solar image is projected. The solar limb is determined at 6 positions every 10 ms, giving pitch and yaw to $\sim 1.5''$. The RAS consists of a star-imaging linear photodiode, providing roll angle to $2.7'$ every minute.

1.2 Expected Imaging Capabilities

In 2-3 years of mission life, HESSI is expected to obtain observations of tens of thousands of microflares, thousands of hard x-ray flares, and of order a hundred gamma-ray line flares. Detected photons will be tagged with 1 μs time resolution up to a rate of $\sim 10^5$ photons/s. The intense 3-150 keV X-ray fluxes that usually accompany large gamma-ray line flares are absorbed by the front segment of the detectors, so the rear segment will always count at moderate rates. This is essential for gamma-ray line measurements with optimal spectral resolution and high throughput.

Normally every photon is stored and then sent down. If an extended extremely active period occurs, the main limitation is the orbit-averaged downlink rate. The GeD rear segments' event rate is always low enough to be stored. When the memory approaches full, the GeD front segments are subsampled by collecting events for an integral number of half-spins for full angular coverage to produce an image, and then not sampling until the input rate matches the expected downlink rate. This method keeps the mass memory from overflowing, with minimum loss of imaging and no loss in gamma-ray spectroscopy.

It is expected that HESSI will provide detailed imaging spectroscopy in each of ten energy intervals with $\sim 2s$ time resolution for one event every ~ 5 days with $\sim 10^4$ counts s^{-1} above 15 keV. With lower energy resolution (~ 100 keV), crude imaging information could be obtained in tens of milliseconds. HESSI provides spatial resolution of 2.3 arcseconds at X-ray energies below ~ 200 keV, 7 arcseconds to 400 keV, and 36 arcseconds for gamma-ray lines and continuum above 1 MeV.

2 Modulation Principles

HESSI, like all RMCs, relies on temporal modulation by its grids to provide spatial information about incoming photons. The imaging process may be thought of as a superposition of elemental processes of photon passage through a sub-collimator. Since the time of arrival of each photon is determined to 1 μs, and the aspect system determines the pointing of the telescope with arcsecond accuracy, the known geometry of the subcollimators provides well-defined information about the photon's origin.

2.1 One-dimensional Modulation

We illustrate how this works by considering a subcollimator with two identical grids, each parallel to the x,y plane, whose slats are, at a given instant, parallel to the y axis, and whose spin lies parallel to the z axis. Let a photon travelling in the x-z plane strike the plane of the front grid. The incidence phase (measured in the x direction) of the photon relative to the slats of the front grid may be considered to be random since all photons arriving from a single point at infinity lie on parallel lines with a continuum of phases, and since the detectors have no spatial resolution, the modulation profile is found by integrating over all incidence phases.

The probability of passage of the photon through the front grid is proportional to the projected area of the slits of that grid, as viewed from the direction of the photon. For an ideal grid of finite thickness and infinite extent, this value is independent of the y direction, but dependent on the photon direction in a fashion characteristic of the grid thickness and the slit-slat ratio.

In the case of a single thin grid with equal slits and slats, the projected area, plotted as a function of incidence phase, is a periodic, rectangular waveform with equally long maxima and minima. For a thick grid, there is internal shadowing by the sides of the slats, and the projected open areas are reduced from the thin case, so the area waveform is still periodic, but with maxima and minima of unequal lengths. In the case of HESSI grids, the projected slit-slat ratios are optimized to have equal projected slit and slat widths at incidence angles of about $600''$. The aspect ratio of the slats (x-width/z-thickness) is 1:50. For photons with nearly normal incidence, the effective slit/pitch ratio is about 0.6, and for photons incident at angles of $415''$ in the x-z plane, the effective slit/pitch ratio after shadowing is close to 1:2.

After photons pass through the front grid, they have a finite probability of passing through the rear grid. If the incidence angle is less than about $900''$, the probability of incidence on the rear grid is almost unity. Only for photons entering at angles of greater than 1 degree does this probability become significantly less than 1 due to non-overlap of the front and rear grid areas. The passage of the photon through the second grid is, again, dependent on its direction in exactly in the same way as for the front grid. The fraction of projected open area for an incident photon is therefore the convolution of the transmission functions of each grid. It is easy to verify that in the ideal case, this is a triangle waveform. For thin grids, the triangle function has pointed peaks and valleys. But if the projected slit-slat ratio is larger or smaller than 1, the valleys of the waveforms will be flattened. The sharp corners of the profile's peaks are also rounded by partial transmission effects.

It is useful and efficient to characterize the subcollimator transmission functions in terms of the first few harmonics. For slit/pitch ratios equal to 1/2, and for energies low enough that the slats are opaque ($E < 100$ keV), and for energies high enough that diffraction effects are negligible ($E > 6$ keV), the effective subcollimator area function is approximately a triangle waveform, represented by the following function of θ, the photon's angle of incidence relative to the imager axis:

$$S_{triangle}(\theta) = A \cdot (1 + \frac{8}{\pi^2} cos(\theta) + \frac{8}{9\pi^2} cos(3\theta) + \frac{8}{25\pi^2} cos(5\theta) + \cdots) \quad (1)$$

But in practice, the 2nd harmonic term is usually not negligible, and the Fourier coefficients are not those of a pure triangle waveform, so we must use the more general form:

$$S(\theta) = A \cdot (1 + C_1 cos[\theta - \theta_1] + C_2 cos(2[\theta - \theta_2]) + C_3 cos(3[\theta - \theta_3]) + \cdots) \quad (2)$$

where A is the effective area averaged over θ. In practice, we keep terms only up to the 5th harmonic. For the case of ideal grids of effective slit/pitch ratio q, the amplitudes C_n are:

$$C_n = (1 - cos[2n\pi q])/(n\pi q)^2 \qquad n = 1, 2, 3, \cdots \quad (3)$$

When $q = 1/2$, the case of a simple triangle profile (which occurs for HESSI when $\theta = 412''$), all the even amplitudes vanish, but for $q = 0.4$, (photons at normal incidence on the HESSI grids), $C_2 = 0.1093$, and for $q = 0.6$ (equivalent to angular incidence of $824''$), $C_2 = 0.0486$. In general, for real conditions, the relative amplitudes $C_1, C_2, \cdots$ and the relative phases $\theta_1, \theta_2, \cdots$ are dependent on the grid characteristics, the roll angle, and the angle of incidence. The latter dependence is sufficiently small and slow that for sources on the Sun, that they can be considered uniform within a map in a given time bin. These parameters have been computed from the grid characterizations, and are included in the HESSI reconstruction algorithms.

2.2 Rotational Modulation

So far we have described the transmission only for one roll angle, namely the instant at which the pattern projected on the sky from an ensemble of sight lines from the detectors – the so-called "modulation pattern" – has contours of constant phase perpendicular to the vector on the plane of the sky from the source to the projected telescope axis. As the spacecraft spins, the projected modulation pattern spins with it. To the lowest order of approximation, if the grids are infinitely wide and infinitely thin, and the spin axis does not move, the modulation pattern would rotate unchanged. Since, however, the grids are thick, of finite extent, and the spin axis is not necessarily constant during a rotation, the projected modulation pattern changes with roll angle in a predictable way, which is reflected in a slow, smooth change of the phase and amplitude coefficients of the sub-collimator pattern.

It is useful to take the telescope as our reference frame, and let a point source move in a circle (roll angle $\alpha = 0{-}2\pi$) on the celestial sphere around the spin axis. Then, because no modulation occurs in the y direction parallel to the slats, and modulation in the x-direction follows equation (2), the effective pitch changing in proportion to $sec(\alpha)$. During a rotation, the fundamental coefficients (C_1, θ_1) change least, and the function S is a simple function of roll angle α. For an ideal RMC (idealized grids, steady spin), the argument θ in equation (3) is proportional to the radial distance R (arcsec) of the source and inversely proportional to angular pitch p):

$$\theta = 2\pi R cos(\alpha)/p \tag{4}$$

The flux of photons on a HESSI detector from a point source a distance R off axis is proportional to the effective area, which leads to the modulation profile as a function of rotation angle α:

$$P_0(\alpha) = A \cdot (1 + C_1 cos(2\pi R cos(\alpha)/p - \theta_1) + \cdots) \tag{5}$$

where p is the angular pitch (arc sec) of the grids in the subcollimator. For a given map center, the quantities (C_n, θ_n) are computed from the aspect system and the grid response matrices, and are returned in the HESSI *calibrated event list.* The above function is computable for any time interval and any point source on the Sun. Therefore, since the the signal expected for a superposition of sources is the superposition of point source signals, any model source can be used to predict a modulation profile. This is the basis for all HESSI reconstruction algorithms.

2.3 The HESSI Event List

The set of photon arrival times taken from the HESSI telemetry can be histogrammed to yield a raw count rate profile. The HESSI user can select an almost arbitrary set of time bin sizes $\{\Delta t_i\}$ for histogramming. (Δt_i is necessarily $\geq 1\ \mu s$, and may be as large as $\sim 1\ s$, but is more typically $\sim 0.5\ ms$). After the time bin selection, the count/bin profile becomes a function of roll angle α, and one can proceed to calibrate the rates.

For phase and amplitude calibration, one must select a phase center for mapping. This point can be found in several ways: (a) determine the azimuth by the roll angle of fastest modulation, and the radius by the equivalent of equation (4); (b) make a rough map of a large region (say a solar quadrant) and find the position of the maximum; or (c) use context data or (d) previous flare position data.

Using the aspect system data, the distance of the line of maximum transmission from map center is computed for each roll angle α_i and each subcollimator pitch p_k. This gives the phase at map center Φ_{ik}. For the case of a fixed spin axis, $\Phi_{ik} = 2\pi R cos(\alpha_i)/p_k - \theta_k$. But in practice, Φ_{ik} is computed without reference to R, using aspect data and the subcollimator geometries alone. One must incorporate the dead time, a function τ_{ij} of roll angle and energy E_j, the detector number k, and the subcollimator transmission T_{ijk}, as well as the modulation amplitudes C_n^{ijk} and modulation phases Φ_n^{ijk} to determine the model modulation profile:

$$S_{ijk} = F \cdot \{\tau_{ijk} T_{ijk} [1 + \sum_n C_n^{ijk} cos[n(\Phi_{ik} - \Phi_n^{ijk})]\} \tag{6}$$

When the HESSI analysis software computes the quantities $\tau_{ijk}, T_{ijk}, C_n^{ijk}$ and Φ_n^{ijk}, they are returned in a structure called the *calibrated event list*, the starting point for image reconstruction.

2.4 Visibilities

In radio astronomy, the quantity analogous to the modulation pattern is the fringe pattern, which is the response of an interferometer to point sources in the sky. This is the Fourier transform, and it is represented by a sum of complex exponentials in the u,v plane:

$$\mathcal{V}_i = f \; e^{2\pi i(u_i x + v_i y)} \tag{7}$$

for each point source of flux f at coordinate (x, y). The variables u_i, v_i are the coordinates in the Fourier plane, and for HESSI, $u_i = cos\phi_i/p$ and $v_i = sin\phi_i/p$, so the points (u_i, v_i) lie on circles in the Fourier plane.
The importance of visibilities to HESSI is that they represent an intermediate between count-rate profiles and images that is independent of the pointing. The visibilities have all of the spinaxis wobble, slit-shadowing effects, transmission and deadtime accounted for. Visibilities, unlike count-rate profiles, can be added from 1/2 rotation to another.

A significant simplification for imaging occurs when one changes from the usual Cartesian coordinates to polar coordinates in both the image plane and the Fourier plane:

$$x = r\ cos\theta \qquad u = k\ cos\phi$$
$$y = r\ sin\theta \qquad v = k\ sin\phi$$

Since the visibilities are found on circles in the Fourier plane, the radial integration drops out ($\mathcal{V}$ is proportional to a delta function of radius k), and after a little algebra, the Fourier transform for one subcollimator of wavenumber k becomes:

$$F(r,\theta) = k \int_0^{2\pi} \mathcal{V}(\phi) e^{ikr\ cos(\theta-\phi)} d\phi = k\mathcal{V} \otimes e^{ikr\ cos(\phi)} \tag{8}$$

The nice thing about this, is that for each radius, $F(r,\theta)$ is a convolution of the visibility and a complex exponential. Since convolutions are orders of magnitude faster than matrix multiplications, polar coordinate representations of HESSI data provide a great increase in speed of backprojection or other reconstructions.

3 Image Reconstruction

3.1 Linear Methods: Backprojection and Fourier Transforms

Back projection (Mertz, Nakano, and Kilner 1986) is the most straightforward and basic method of image reconstruction. It is equivalent to a Fourier transform (Kilner and Nakano, 1989.) A map constructed by this method is called the "dirty map", being the analogue of the radio astronomer's initial Fourier transform of the observed visibilities. Back projection is a perfectly linear process: maps for arbitrary time intervals may be added together, and maps for different pitches and harmonics may be summed, generally leading to improvement of the image. Further improvements to the image by CLEAN or MEM (for example) do not share this property of linearity.

The principle of back projection is to consider the quantities S_{ijk} in equation (6) as circular slices of the two-dimensional modulation patterns. The modulation profile (6) may be thought of as the amplitude at a given radius and fixed azimuth of a series of two-dimensional modulation patterns that rotate around at the same rate as the telescope. The modulation patterns must shift in phase and amplitude, depending on the relative position of the telescope axis as a function of roll angle.

$$M_{ijk}(x,y) = F \cdot \{\tau_{ijk} T_{ijk}[1+ \sum_n C_n^{ijk} cos[2\pi n[(x-x_c)cos\alpha + \quad (y-y_c)sin\alpha] + \Phi_{ik} - \Phi_n^{ijk})]\}$$

Although it is possible to use only one modulation pattern per roll angle to produce back-projection images, it turns out to be more efficient to construct "universal" modulation patterns in the form of cosine and sine pairs. Sinusoidal modulation patterns of arbitrary phase can then be computed by appropriate linear combinations. This formulation has the advantage that neighboring half rotations may use the same "universal" set of patterns, even if there are different aspect solution profiles.

It is worth mentioning that in early versions of RMC design, there were both sine and cosine subcollimators (*e.g.*, Mertz *et al.* 1986), Murphy, 1990), but this turns out not to be necessary in hardware, since with sufficient sampling of the modulation pattern, relative amplitudes of the sine and cosine components can be determined from the modulation profile itself.

Each modulation pattern has contours of constant phase perpendicular to the roll angle azimuth vector, and has the periods and amplitudes used in equation (6). The modulation patterns so computed for each angle bin, are added together with weights given by the count rate divided by the product of the transmission factors and deadtimes. The sum is the back-projection map. For HESSI, simulations suggest that summation of the 9-subcollimator back-projection maps gives a dynamic range of order 10:1 for rates $\sim 10^5 counts^{-1} s^{-1}$.

3.2 Nonlinear Reconstruction Methods

All of the practical image reconstruction algorithms for improving a back-projection ("dirty") map are nonlinear, since linear deconvolution algorithms such as Wiener filtering and inverse filtering are inapplicable to applications with incomplete sampling of the Fourier plane, as is the case for HESSI modulation synthesis. A number of reconstruction methods already exist, or are being improved, or are being considered for addition to the HESSI reconstruction suite. We summarize several of these here.

3.2.1 CLEAN (Cartesian, Polar)

CLEAN is an iterative algorithm which deconvolves a Point Spread Function–the imager's response to a delta function source–from a "dirty map". This algorithm is of fundamental importance in radio astronomy, where it is used to create images of astronomical sources obtained with interferometers. The basic CLEAN method was developed by Högbom (1974). It was originally designed for point sources, but it has been found to work well for extended sources as well when given a reasonable starting model. The Högbom CLEAN constructs discrete approximations to the CLEAN Map in the plane from the convolution equation

$$\mathcal{P} \otimes I_{source} = D \tag{9}$$

where P is the HESSI PSFfor one or many subcollimators and/or harmonics, D is the dirty map, and $\otimes$ denotes a convolution. The CLEAN algorithm starts with an initial approximation $I_0 = 0$ to the residual map. At the n^{th} iteration, it then searches for the largest value in the residual map. A delta function is then centered at the location of the largest residual flux and given an amplitude μ (the so-called "loop gain") times this value. P, is then subtracted from I_{n-1} to yield I_n.

$$I_n = I_{n-1} \ - \ \mu\mathcal{P} \tag{10}$$

Iteration continues until a specified iteration limit is reached, or until the peak residual or RMS residual decreases to some level. The resulting final map is denoted I_{final}, and the position of each delta function is saved in a "CLEAN component" table. At the point where component subtraction is stopped, it is assumed that the residual brightness distribution consists mainly of noise.

To damp out high spatial frequency features which may be spuriously created in the iteration, each CLEAN component is convolved with the so-called CLEAN PSF, $\mathcal{P}_{clean}$ (the "CLEAN beam" in radioastronomy), which is simply a suitably smoothed version or inner portion of the PSF, $\mathcal{P}$. A CLEAN map is produced when the final residual map is added to the approximate solution,

$$I_{clean} = I_{final} \otimes \mathcal{P}_{clean} \ + \ \{D - \mathcal{P} \otimes I_{final}\} \tag{11}$$

in order to include the noise.

The main disadvantage of CLEAN is that it does not, at least in the Högbom version, compare the observed modulation profile with a model modulation profile, to assess the "goodness of fit" during the iteration. Two variants of CLEAN exist for HESSI. One is for rectangular coordinates, and the other is for polar coordinates. In most circumstances, the latter has a significant advantage of speed over the former.

3.2.2 Maximum Entropy Methods (MEM Sato, Polar MEM)

The principle of Maximum Entropy is to find the map which is maximally noncommittal with regard to the unavailable information (*i.e.* the regions of the Fourier plane for which HESSI has no data). The image found is that which fits the data to within the noise level and also maximizes the entropy. In one view, the entropy is something which, when maximized, produces a positive image with a compressed range in pixel values. (Cornwell 1984)Image entropy thus defined is therefore not to be confused with a "physical entropy". Several kinds of "entropy" have been proposed to achieve this, but according to Skilling (1984) and Sivia (1996) the only function which guarantees that no untoward correlation is imposed is:

$$\mathcal{H} \; = -\sum_{j=1}^{M} f_j \; ln(f_j) \tag{12}$$

The idea is to maximize the objective function Q:

$$Q \; = \; \mathcal{H} \; - \lambda \, \chi^2 \; - \; \beta \sum_{1}^{M} f_j \tag{13}$$

for some values of λ, where λ is a parameter giving a measure of the tug-of-war between maximizing entropy and minimizing the χ^2 statistic and β is a Lagrange parameter multiplying the flux constraint. At maximum,

$$f_j = F - j \; e^{-\lambda \partial \chi^2 / \partial f_j - \alpha} \tag{14}$$

One may solve the equations (11) iteratively or by directed search of Q.

3.2.3 Forward Fitting (pixelized, non-pixelized)

The forward-fitting method is based on models that represent a spatial map by a superposition of multiple source structures, which are quantified by circular gaussians (4 parameters per source), elliptical gaussians (6 parameters per source), or curved ellipticals (7 parameters), designed to characterize real solar flare hard X-ray maps with a minimum number of geometric elements.

In one realization of this scheme, maps of superimposed Gaussians are created iteratively, at each step constructing a model modulation profile from the map. In this "pixelized" form, the map evolves into a best fitting image. A second form of forward-fitting is the unpixelized version, where the parameters of the Gaussians are used to find the "equivalent point source", which is equivalent to an amplitude and phase at each roll angle. The "equivalent point source" is then used to create the model modulation profile for comparison with the observed count profile and continued iteration. The unpixelized form requires at least an order of magnitude less memory and computation than the pixelized form, but the programs are more complex and their robustness is as yet unknown.

3.2.4 The PIXON Method

The PIXON method is another technique removes the sidelobe pattern of a telescope while mitigating the problems of correlated residuals and spurious sources which are commonly seen in Fourier deconvolution, chi-square fitting, and Maximum Entropy approaches.

The goal of the Pixon method is to construct the simplest, i.e. smoothest, model for the image that would be consistent with the data, i.e. have an acceptable chi-square fit. Being the simplest model, the derived image would be artifact free, i.e. there would be no spurious sources. In addition, the model is necessarily be most tightly constrained by the data, and consequently have the most accurately determined parameters.

The PIXON method changes the global smoothness idea of MEM into a local condition in which local maximum smoothness of the image is imposed. From an information science point of view, one selects a model with the minimum information content from the family of multiresolution basis functions (pixons) and which statistically fit the data.

Since the model has minimum complexity, spurious sources are unlikely to arise. Each parameter is determined using a larger fraction of the data, hence it is determined more accurately. This usually results in superior photometric and positional accuracy. And since the minimum number of parameters are used, the data cannot be over fitted.

3.3 Undeveloped Techniques (Wavelets, NNLS)

HESSI has pitches at multiplicative intervals of $\sqrt{3}$, and so, in a way it is a wavelet telescope, where the wavelets are at scales logarithmically spaced by $\sqrt{3}$. To date, no one has exploited wavelet mathematics for HESSI imaging.

A promising technique used for medical imaging (ref) is Non-Negative Least Squares (NNLS). In general, linear Least Squares is not expected to be useful for HESSI, but the enforcement of positivity in NNLS provides an essential nonlinear constraint that may lead to good images in the HESSI parameter domain.

4 Statistical Limits

Since HESSI is a photon-counting instrument, imaging involves stochastic processes. During the time during which the data for an image are acquired, the incident photon rate λ may be considered a constant. The number of photons per time bin is given by Poisson statistics, where the probability density function $p_{poi}(\lambda)$ is the expected number of photons per time bin incident on the front grid. The process of modulation is also stochastic, since the incident phase relative to the collimator fringes is random, and the probability of a photon detection is proportional to the normalized area function (6). The counts per bin is therefore given by the product of these two probabilities:

$$\mathcal{C}_i = p_{poi}(\lambda)\ S_{norm} \tag{15}$$

Examination of equation (6) for S_{ijk} shows that if its Fourier expansion is truncated at the fundamental (as is often very useful), there are situations when S can go negative. This is not really a problem for backprojection or for Fourier mapping, neither of which depend on positivity. (And possibly also not for Forward Fitting.) But it is of importance for CLEAN, MEM, and Pixons. Of singular importance, if it is negative, S_{norm} in equation (16) fails to have any meaning as a probability.

The problem arises due to an expectation that S_{ijk} should represent a count rate. If, however, the Fourier coefficients are considered as primary elements, then the statistics can be done with them, regardless of their number and whether they produce a negative sum.

Let us consider the fundamental only. After choosing a map center, at any given rotation angle, one can use the calibrated phase at map center to determine the angular length of a modulation cycle. For the fundamental, the Fourier coefficients can be computed from 4 angular bins, assumed to be equally spaced within one cycle. Let these bins have counts $\{s_0, s_1, s_2, s_3\}$. Then the DC, cosine and sine coeffcients may be written as:

$$
\begin{aligned}
C_0 &= [s_0 + s_1 + s_2 + s_3]/4 \\
C_1^{cos} &= [s_0 \; - \; s_2]/4 \\
C_1^{sin} &= [s_1 \; - \; s_3]/4
\end{aligned}
$$

Then, since $\sigma^2 = \lambda$ for Poisson statistics, the standard deviation for the DC coefficient is, as expected, $\sigma_0 = \sqrt{\lambda}$, and for the fundamental, sine and cosine, $\sigma_1 = \sqrt{\lambda/2}$. A similar analysis can be done for the higher harmonics $(n > 1)$, where of course it is necessary to have n bins per fundamental cycle for adequate sampling. For the nth harmonic, the standard deviation of the sine and cosine coefficients is $\sigma_n = \sqrt{\lambda/2n}$.

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